Fundamentals of Die Casting Design

Genick Bar–Meir, Ph. D.
7449 North Washtenaw Ave
Chicago, IL 60645
email:barmeir at gmail dot com

Version (0.1.4 November 27, 2012)

‘We are like dwarfs sitting on the shoulders of giants”

from The Metalogicon by John in 1159

CONTENTS

LIST OF FIGURES

LIST OF TABLES

NOMENCLATURE

$\bar{R}$ Universal gas constant, see equation (2.36), page 18

ℓ Units length., see equation (2.11), page 14

ρ Density of the fluid, see equation (2.55), page 22

B bulk modulus, see equation (2.62), page 22

B_f Body force, see equation (2.19), page 15

c Speed of sound, see equation (2.55), page 22

C_p Specific pressure heat, see equation (2.33), page 17

C_v Specific volume heat, see equation (2.32), page 17

E_U Internal energy, see equation (2.13), page 15

E_u Internal Energy per unit mass, see equation (2.16), page 15

E_i System energy at state i, see equation (2.12), page 14

H Enthalpy, see equation (2.28), page 17

h Specific enthalpy, see equation (2.28), page 17

k the ratio of the specific heats, see equation (2.34), page 18

M Mach number, see equation (2.64), page 23

n The poletropic coefficient, see equation (2.60), page 22

P Pressure, see equation (2.57), page 22

q Energy per unit mass, see equation (2.16), page 15

Q_{12} The energy transferred to the system between state 1 and state 2, see equation (2.12), page 14

R Specific gas constant, see equation (2.37), page 18

S Entropy of the system, see equation (2.23), page 16

U velocity , see equation (2.14), page 15

w Work per unit mass, see equation (2.16), page 15

W_{12} The work done by the system between state 1 and state 2, see equation (2.12), page 14

The Book Change Log

Version 0.1.4

Nov 27, 2012 (1.9M 269 pages)

- Additional discussion on the economics chapter on.

Version 0.1.3

Nov 8, 2012 (1.9M 265 pages)

- Improvements to some of the figures of dimensional analysis chapter (utilizing blender).
- Add an analysis of the minimum cost ordering supply. The minimum cost ordering refers to the analysis dealing with the minimum cost achieved by finding the optimum number of ordering.

Version 0.1.2

April 1, 2009 (1.9M 263 pages)

- Irene Tan provided many English corrections to the dimensional analysis chapter.

Version 0.1.1

Feb 8, 2009 (1.9M 261 pages)

- Add Steve Spurgeon (from Dynacast England) corrections to pQ^2 diagram.

- Minor English corrections to pQ^2 diagram chapter (unfinished).
- Fix some figures and captions issues.
- Move to potto style file.

Version 0.1

Jan 6, 2009 (1.6M 213 pages)

- Change to modern Potto format.
- English corrections
- Finish some examples in Dimensionless Chapter (manometer etc)

Version 0.0.3

Nov 1, 1999 (3.1 M 178 pages)

- Initial book of Potto project.
- Start of economy, dimensional analysis, pQ^2 diagram chapters.

Notice of Copyright For This Document:

This document published Modified FDL. The change of the license is to prevent from situations where the author has to buy his own book. The Potto Project License isn't long apply to this document and associated docoments.

GNU Free Documentation License

The modification is that under section 3 "copying in quantity" should be add in the end.

"If you print more than 200 copies, you are required to furnish the author with two (2) copies of the printed book."

Preamble

The purpose of this License is to make a manual, textbook, or other functional and useful document "free" in the sense of freedom: to assure everyone the effective freedom to copy and redistribute it, with or without modifying it, either commercially or noncommercially. Secondarily, this License preserves for the author and publisher a way to get credit for their work, while not being considered responsible for modifications made by others.

This License is a kind of "copyleft", which means that derivative works of the document must themselves be free in the same sense. It complements the GNU General Public License, which is a copyleft license designed for free software.

We have designed this License in order to use it for manuals for free software, because free software needs free documentation: a free program should come with manuals providing the same freedoms that the software does. But this License is not limited to software manuals; it can be used for any textual work, regardless of subject matter or whether it is published as a printed book. We recommend this License principally for works whose purpose is instruction or reference.

1. APPLICABILITY AND DEFINITIONS

This License applies to any manual or other work, in any medium, that contains a notice placed by the copyright holder saying it can be distributed under the terms of this License. Such a notice grants a world-wide, royalty-free license, unlimited in duration, to use that work under the conditions stated herein. The **"Document"**, below, refers to any such manual or work. Any member of the public is a licensee, and is addressed as **"you"**. You accept the license if you copy, modify or distribute the work in a way requiring permission under copyright law.

A **"Modified Version"** of the Document means any work containing the Document or a portion of it, either copied verbatim, or with modifications and/or translated into another language.

A **"Secondary Section"** is a named appendix or a front-matter section of the Document that deals exclusively with the relationship of the publishers or authors of the Document to the Document's overall subject (or to related matters) and contains nothing that could fall directly within that overall subject. (Thus, if the Document is in part a textbook of mathematics, a Secondary Section may not explain any mathematics.) The relationship could be a matter of historical connection with the subject or with related matters, or of legal, commercial, philosophical, ethical or political position regarding them.

The **"Invariant Sections"** are certain Secondary Sections whose titles are designated, as being those of Invariant Sections, in the notice that says that the Document is released under this License. If a section does not fit the above definition of Secondary then it is not allowed to be designated as Invariant. The Document may contain zero Invariant Sections. If the Document does not identify any Invariant Sections then there are none.

The **"Cover Texts"** are certain short passages of text that are listed, as Front-Cover Texts or Back-Cover Texts, in the notice that says that the Document is released under this License. A Front-Cover Text may be at most 5 words, and a Back-Cover Text may be at most 25 words.

A **"Transparent"** copy of the Document means a machine-readable copy, represented in a format whose specification is available to the general public, that is suitable for revising the document straightforwardly with generic text editors or (for images composed of pixels) generic paint programs or (for drawings) some widely available drawing editor, and that is suitable for input to text formatters or for automatic translation to a variety of formats suitable for input to text formatters. A copy made in an otherwise Transparent file format whose markup, or absence of markup, has been arranged to thwart or discourage subsequent modification by readers is not Transparent. An image format is not Transparent if used for any substantial amount of text. A copy that is not "Transparent" is called **"Opaque"**.

Examples of suitable formats for Transparent copies include plain ASCII without markup, Texinfo input format, LaTeX input format, SGML or XML using a publicly available DTD, and standard-conforming simple HTML, PostScript or PDF designed for human modification. Examples of transparent image formats include PNG, XCF and JPG. Opaque formats include proprietary formats that can be read and edited only by proprietary word processors, SGML or XML for which the DTD and/or processing tools are not generally available, and the machine-generated HTML, PostScript or PDF produced by some word processors for output purposes only.

The **"Title Page"** means, for a printed book, the title page itself, plus such following pages as are needed to hold, legibly, the material this License requires to appear in the title page. For works in formats which do not have any title page as such, "Title Page" means the text near the most prominent appearance of the work's title, preceding the beginning of the body of the text.

A section **"Entitled XYZ"** means a named subunit of the Document whose title either is precisely XYZ or contains XYZ in parentheses following text that translates XYZ in another language. (Here XYZ stands for a specific section name mentioned below, such as **"Acknowledgements"**, **"Dedications"**, **"Endorsements"**, or **"History"**.) To **"Preserve the Title"** of such a section when you modify the Document means that it remains a section "Entitled XYZ" according to this definition.

The Document may include Warranty Disclaimers next to the notice which states that this License applies to the Document. These Warranty Disclaimers are considered to be included by reference in this License, but only as regards disclaiming warranties: any other implication that these Warranty Disclaimers may have is void and has no effect on the meaning of this License.

2. VERBATIM COPYING

You may copy and distribute the Document in any medium, either commercially or noncommercially, provided that this License, the copyright notices, and the license notice saying this License applies to the Document are reproduced in all copies, and that you add no other conditions whatsoever to those of this License. You may not use technical measures to obstruct or control the reading or further copying of the copies you make or distribute. However, you may accept compensation in exchange for copies. If you distribute a large enough number of copies you must also follow the conditions in section 3.

You may also lend copies, under the same conditions stated above, and you may publicly display copies.

3. COPYING IN QUANTITY

If you publish printed copies (or copies in media that commonly have printed covers) of the Document, numbering more than 100, and the Document's license notice requires Cover Texts, you must enclose the copies in covers that carry, clearly and legibly, all these Cover Texts: Front-Cover Texts on the front cover, and Back-Cover Texts on the back cover. Both covers must also clearly and legibly identify you as the publisher of these copies. The front cover must present the full title with all words of the title equally prominent and visible. You may add other material on the covers in addition. Copying with changes limited to the covers, as long as they preserve the title of the Document and satisfy these conditions, can be treated as verbatim copying in other respects.

If the required texts for either cover are too voluminous to fit legibly, you should put the first ones listed (as many as fit reasonably) on the actual cover, and continue the rest onto adjacent pages.

If you publish or distribute Opaque copies of the Document numbering more than 100, you must either include a machine-readable Transparent copy along with each Opaque copy, or state in or with each Opaque copy a computer-network location from which the general network-using public has access to download using public-standard network protocols a complete Transparent copy of the Document, free of added material. If you use the latter option, you must take reasonably prudent steps, when you begin distribution of Opaque copies in quantity, to ensure that this Transparent copy will remain thus accessible at the stated location until at least one year after the last time you distribute an Opaque copy (directly or through your agents or retailers) of that edition to the public.

It is requested, but not required, that you contact the authors of the Document well before redistributing any large number of copies, to give them a chance to provide you with an updated version of the Document.

4. MODIFICATIONS

You may copy and distribute a Modified Version of the Document under the conditions of sections 2 and 3 above, provided that you release the Modified Version under precisely this License, with the Modified Version filling the role of the Document, thus licensing distribution and modification of the Modified Version to whoever possesses a copy of it. In addition, you must do these things in the Modified Version:

A. Use in the Title Page (and on the covers, if any) a title distinct from that of the Document, and from those of previous versions (which should, if there were any, be listed in the History section of the Document). You may use the same title as a previous version if the original publisher of that version gives permission.

B. List on the Title Page, as authors, one or more persons or entities responsible for authorship of the modifications in the Modified Version, together with at least five of the principal authors of the Document (all of its principal authors, if it has fewer than five), unless they release you from this requirement.

C. State on the Title page the name of the publisher of the Modified Version, as the publisher.

D. Preserve all the copyright notices of the Document.

E. Add an appropriate copyright notice for your modifications adjacent to the other copyright notices.

F. Include, immediately after the copyright notices, a license notice giving the public permission to use the Modified Version under the terms of this License, in the form shown in the Addendum below.

G. Preserve in that license notice the full lists of Invariant Sections and required Cover Texts given in the Document's license notice.

H. Include an unaltered copy of this License.

I. Preserve the section Entitled "History", Preserve its Title, and add to it an item stating at least the title, year, new authors, and publisher of the Modified Version as given on the Title Page. If there is no section Entitled "History" in the Document, create one stating the title, year, authors, and publisher of the Document as given on its Title Page, then add an item describing the Modified Version as stated in the previous sentence.

J. Preserve the network location, if any, given in the Document for public access to a Transparent copy of the Document, and likewise the network locations given in the Document for previous versions it was based on. These may be placed in the "History" section. You may omit a network location for a work that was published at least four years before the Document itself, or if the original publisher of the version it refers to gives permission.

K. For any section Entitled "Acknowledgements" or "Dedications", Preserve the Title of the section, and preserve in the section all the substance and tone of each of the contributor acknowledgements and/or dedications given therein.

L. Preserve all the Invariant Sections of the Document, unaltered in their text and in their titles. Section numbers or the equivalent are not considered part of the section titles.

M. Delete any section Entitled "Endorsements". Such a section may not be included in the Modified Version.

N. Do not retitle any existing section to be Entitled "Endorsements" or to conflict in title with any Invariant Section.

O. Preserve any Warranty Disclaimers.

If the Modified Version includes new front-matter sections or appendices that qualify as Secondary Sections and contain no material copied from the Document, you may at your option designate some or all of these sections as invariant. To do this, add their titles to the list of Invariant Sections in the Modified Version's license notice. These titles must be distinct from any other section titles.

You may add a section Entitled "Endorsements", provided it contains nothing but endorsements of your Modified Version by various parties–for example, statements of peer review or that the text has been approved by an organization as the authoritative definition of a standard.

You may add a passage of up to five words as a Front-Cover Text, and a passage of up to 25 words as a Back-Cover Text, to the end of the list of Cover Texts in the Modified Version. Only one passage of Front-Cover Text and one of Back-Cover Text may be added by (or through arrangements made by) any one entity. If the Document already includes a cover text for the same cover, previously added by you or by arrangement made by the same entity you are acting on behalf of, you may not add another; but you may replace the old one, on explicit permission from the previous publisher that added the old one.

The author(s) and publisher(s) of the Document do not by this License give permission to use their names for publicity for or to assert or imply endorsement of any Modified Version.

5. COMBINING DOCUMENTS

You may combine the Document with other documents released under this License, under the terms defined in section 4 above for modified versions, provided that you include in the combination all of the Invariant Sections of all of the original documents, unmodified, and list them all as Invariant Sections of your combined work in its license notice, and that you preserve all their Warranty Disclaimers.

The combined work need only contain one copy of this License, and multiple identical Invariant Sections may be replaced with a single copy. If there are multiple Invariant Sections with the same name but different contents, make the title of each such section unique by adding at the end of it, in parentheses, the name of the original author or publisher of that section if known, or else a unique number. Make the same adjustment to the section titles in the list of Invariant Sections in the license notice of the combined work.

In the combination, you must combine any sections Entitled "History" in the various original documents, forming one section Entitled "History"; likewise combine any sections Entitled "Acknowledgements", and any sections Entitled "Dedications". You must delete all sections Entitled "Endorsements".

6. COLLECTIONS OF DOCUMENTS

You may make a collection consisting of the Document and other documents released under this License, and replace the individual copies of this License in the various documents with a single copy that is included in the collection, provided that you follow the rules of this License for verbatim copying of each of the documents in all other respects.

You may extract a single document from such a collection, and distribute it individually under this License, provided you insert a copy of this License into the extracted document, and follow this License in all other respects regarding verbatim copying of that document.

7. AGGREGATION WITH INDEPENDENT WORKS

A compilation of the Document or its derivatives with other separate and independent documents or works, in or on a volume of a storage or distribution medium, is called an "aggregate" if the copyright resulting from the compilation is not used to limit the legal rights of the compilation's users beyond what the individual works permit. When the Document is included in an aggregate, this License does not apply to the other works in the aggregate which are not themselves derivative works of the Document.

If the Cover Text requirement of section 3 is applicable to these copies of the Document, then if the Document is less than one half of the entire aggregate, the Document's Cover Texts may be placed on covers that bracket the Document within the aggregate, or the electronic equivalent of covers if the Document is in electronic form. Otherwise they must appear on printed covers that bracket the whole aggregate.

8. TRANSLATION

Translation is considered a kind of modification, so you may distribute translations of the Document under the terms of section 4. Replacing Invariant Sections with translations requires special permission from their copyright holders, but you may include translations of some or all Invariant Sections in addition to the original versions of these Invariant Sections. You may include a translation of this License, and all the license notices in the Document, and any Warranty Disclaimers, provided that you also include the original English version of this License and the original versions of those notices and disclaimers. In case of a disagreement between the translation and the original version of this License or a notice or disclaimer, the original version will prevail.

If a section in the Document is Entitled "Acknowledgements", "Dedications", or "History", the requirement (section 4) to Preserve its Title (section 1) will typically require changing the actual title.

9. TERMINATION

You may not copy, modify, sublicense, or distribute the Document except as expressly provided for under this License. Any other attempt to copy, modify, sublicense or distribute the Document is void, and will automatically terminate your rights under this License. However, parties who have received copies, or rights, from you under this License will not have their licenses terminated so long as such parties remain in full compliance.

10. FUTURE REVISIONS OF THIS LICENSE

The Free Software Foundation may publish new, revised versions of the GNU Free Documentation License from time to time. Such new versions will be similar in spirit to the present version, but may differ in detail to address new problems or concerns. See http://www.gnu.org/copyleft/.

Each version of the License is given a distinguishing version number. If the Document specifies that a particular numbered version of this License "or any later version" applies to it, you have the option of following the terms and conditions either of that specified version or of any later version that has been published (not as a draft) by the Free Software Foundation. If the Document does not specify a version number of this License, you may choose any version ever published (not as a draft) by the Free Software Foundation.

ADDENDUM: How to use this License for your documents

To use this License in a document you have written, include a copy of the License in the document and put the following copyright and license notices just after the title page:

> Copyright ©YEAR YOUR NAME. Permission is granted to copy, distribute and/or modify this document under the terms of the GNU Free Documentation License, Version 1.2 or any later version published by the Free Software Foundation; with no Invariant Sections, no Front-Cover Texts, and no Back-Cover Texts. A copy of the license is included in the section entitled "GNU Free Documentation License".

If you have Invariant Sections, Front-Cover Texts and Back-Cover Texts, replace the "with...Texts." line with this:

> with the Invariant Sections being LIST THEIR TITLES, with the Front-Cover Texts being LIST, and with the Back-Cover Texts being LIST.

If you have Invariant Sections without Cover Texts, or some other combination of the three, merge those two alternatives to suit the situation.

If your document contains nontrivial examples of program code, we recommend releasing these examples in parallel under your choice of free software license, such as the GNU General Public License, to permit their use in free software.

CONTRIBUTORS LIST

How to contribute to this book

As a copylefted work, this book is open to revisions and expansions by any interested parties. The only "catch" is that credit must be given where credit is due. This *is* a copyrighted work: it is *not* in the public domain!

If you wish to cite portions of this book in a work of your own, you must follow the same guidelines as for any other GDL copyrighted work.

Credits

All entries have been arranged in alphabetical order of surname (hopefully). Major contributions are listed by individual name with some detail on the nature of the contribution(s), date, contact info, etc. Minor contributions (typo corrections, etc.) are listed by name only for reasons of brevity. Please understand that when I classify a contribution as "minor," it is in no way inferior to the effort or value of a "major" contribution, just smaller in the sense of less text changed. Any and all contributions are gratefully accepted. I am indebted to all those who have given freely of their own knowledge, time, and resources to make this a better book!

- **Date(s) of contribution(s):** 1999 to present
- **Nature of contribution:** Original author.
- **Contact at:** `barmeir at gmail.com`

Steven from artofproblemsolving.com

- **Date(s) of contribution(s):** June 2005
- **Nature of contribution:** LaTeX formatting, help on building the useful equation and important equation macros.

Tousher Yang

- **Date(s) of contribution(s):** Mat 2008
- **Nature of contribution:** Major review of dimensional analysis and intro chapters.

Steve Spurgeon

- **Date(s) of contribution(s):** November 200x
- **Nature of contribution:** Correction to pQ^2 diagram derivations.

Irene Tan

- **Date(s) of contribution(s):** January, 2009
- **Nature of contribution:** Repair of dimensional analysis chapter.

Your name here

- **Date(s) of contribution(s):** Month and year of contribution
- **Nature of contribution:** Insert text here, describing how you contributed to the book.
- **Contact at:** `my_email@provider.net`

Typo corrections and other "minor" contributions

- **John Joansson** English corrections 1999
- **Adeline Ong** English corrections 1999
- **Robert J. Fermin** English corrections 1999
- **Mary Fran Riley** English corrections 1999
- **Joy Branlund** English corrections 1999
- **Denise Pfeifer** English corrections 1999
- **F. Montery**, point to typos in the book 2000.
- **Irene Tan**, English correction to Fluid Mechanics chapter 2009.

About This Author

Genick Bar-Meir holds a Ph.D. in Mechanical Engineering from University of Minnesota and a Master in Fluid Mechanics from Tel Aviv University. Dr. Bar-Meir was the last student of the late Dr. R.G.E. Eckert. Much of his time has been spend doing research in the field of heat and mass transfer (related to renewal energy issues) and this includes fluid mechanics related to manufacturing processes and design. Currently, he spends time writing books (there are already three very popular books) and softwares for the POTTO project (see Potto Prologue). The author enjoys to encourage his students to understand the material beyond the basic requirements of exams.

Bar-Meir's books are used by hundred of thousands of peoples. His book on compressible is the most popular and preferred by practitioners and students. His books books are used in many universities like Purdue, Caltech, Queens University in Canada, and Singapore. One reason that his books are so popular is that they contain up to date material much of it original work by Bar-Meir.

In his early part of his professional life, Bar-Meir was mainly interested in elegant models whether they have or not a practical applicability. Now, this author's views had changed and the virtue of the practical part of any model becomes the essential part of his ideas, books and software. He developed models for Mass Transfer in high concentration that became a building blocks for many other models. These models are based on analytical solution to a family of equations[1]. As the change in the view occurred, Bar-Meir developed models that explained several manufacturing processes such the rapid evacuation of gas from containers, the critical piston velocity in a partially filled chamber (related to hydraulic jump), application of supply and demand to rapid change power system and etc. All the models have practical applicability. These models have been extended by several research groups (needless to say with large research grants). For example, the Spanish Comision Interministerial provides grants TAP97-0489 and PB98-0007, and the CICYT and the European Commission provides

[1]Where the mathematicians were able only to prove that the solution exists.

1FD97-2333 grants for minor aspects of that models. Moreover, the author's models were used in numerical works, in GM, British industry, Spain, and Canada.

In the area of compressible flow, it was commonly believed and taught that there is only weak and strong shock and it is continue by Prandtl–Meyer function. Bar–Meir discovered the analytical solution for oblique shock and showed that there is a quiet buffer between the oblique shock and Prandtl–Meyer. He also build analytical solution to several moving shock cases. He described and categorized the filling and evacuating of chamber by compressible fluid in which he also found analytical solutions to cases where the working fluid was ideal gas. The common explanation to Prandtl–Meyer function shows that flow can turn in a sharp corner. Engineers have constructed design that based on this conclusion. Bar-Meir demonstrated that common Prandtl–Meyer explanation violates the conservation of mass and therefor the turn must be around a finite radius. The author's explanations on missing diameter and other issues in fanno flow and ““naughty professor's question”” are used in the industry.

In his book "Basics of Fluid Mechanics", Bar-Meir demonstrated that fluids must have wavy surface when two different materials flow together. All the previous models for the flooding phenomenon did not have a physical explanation to the dryness. He built a model to explain the flooding problem (two phase flow) based on the physics. He also constructed and explained many new categories for two flow regimes.

The author lives with his wife and three children. A past project of his was building a four stories house, practically from scratch. While he writes his programs and does other computer chores, he often feels clueless about computers and programing. While he is known to look like he knows about many things, the author just know to learn quickly. The author spent years working on the sea (ships) as a engine sea officer but now the author prefers to remain on solid ground.

Prologue For The POTTO Project

Preface

This books series was born out of frustrations in two respects. The first issue is the enormous price of college textbooks. It is unacceptable that the price of the college books will be over $150 per book (over 10 hours of work for an average student in The United States).

The second issue that prompted the writing of this book is the fact that we as the public have to deal with a corrupted judicial system. As individuals we have to obey the law, particularly the copyright law with the "infinite[2]" time with the copyright holders. However, when applied to "small" individuals who are not able to hire a large legal firm, judges simply manufacture facts to make the little guy lose and pay for the defense of his work. On one hand, the corrupted court system defends the "big" guys and on the other hand, punishes the small "entrepreneur" who tries to defend his or her work. It has become very clear to the author and founder of the POTTO Project that this situation must be stopped. Hence, the creation of the POTTO Project. As R. Kook, one of this author's sages, said instead of whining about arrogance and incorrectness, one should increase wisdom. This project is to increase wisdom and humility.

The POTTO Project has far greater goals than simply correcting an abusive Judicial system or simply exposing abusive judges. It is apparent that writing textbooks especially for college students as a cooperation, like an open source, is a new idea[3]. Writing a book in the technical field is not the same as writing a novel. The writing of a technical book is really a collection of information and practice. There is always someone who can add to the book. The study of technical material isn't only done by having to memorize the material, but also by coming to understand and be able to solve related problems. The author has not found any technique that is more useful for this

[2]After the last decision of the Supreme Court in the case of Eldred v. Ashcroff (see `http://cyber.law.harvard.edu/openlaw/eldredvashcroft` for more information) copyrights practically remain indefinitely with the holder (not the creator).

[3]In some sense one can view the encyclopedia Wikipedia as an open content project (see `http://en.wikipedia.org/wiki/Main_Page`). The wikipedia is an excellent collection of articles which are written by various individuals.

purpose than practicing the solving of problems and exercises. One can be successful when one solves as many problems as possible. To reach this possibility the collective book idea was created/adapted. While one can be as creative as possible, there are always others who can see new aspects of or add to the material. The collective material is much richer than any single person can create by himself.

The following example explains this point: The army ant is a kind of carnivorous ant that lives and hunts in the tropics, hunting animals that are even up to a hundred kilograms in weight. The secret of the ants' power lies in their collective intelligence. While a single ant is not intelligent enough to attack and hunt large prey, the collective power of their networking creates an extremely powerful intelligence to carry out this attack[4]. When an insect which is blind can be so powerful by networking, So can we in creating textbooks by this powerful tool.

Why Volunteer?

Why would someone volunteer to be an author or organizer of such a book? This is the first question the undersigned was asked. The answer varies from individual to individual. It is hoped that because of the open nature of these books, they will become the most popular books and the most read books in their respected field. For example, the books on compressible flow and die casting became the most popular books in their respective area. In a way, the popularity of the books should be one of the incentives for potential contributors. The desire to be an author of a well–known book (at least in his/her profession) will convince some to put forth the effort. For some authors, the reason is the pure fun of writing and organizing educational material. Experience has shown that in explaining to others any given subject, one also begins to better understand the material. Thus, contributing to these books will help one to understand the material better. For others, the writing of or contributing to this kind of books will serve as a social function. The social function can have at least two components. One component is to come to know and socialize with many in the profession. For others the social part is as simple as a desire to reduce the price of college textbooks, especially for family members or relatives and those students lacking funds. For some contributors/authors, in the course of their teaching they have found that the textbook they were using contains sections that can be improved or that are not as good as their own notes. In these cases, they now have an opportunity to put their notes to use for others. Whatever the reasons, the undersigned believes that personal intentions are appropriate and are the author's/organizer's private affair.

If a contributor of a section in such a book can be easily identified, then that contributor will be the copyright holder of that specific section (even within question/answer sections). The book's contributor's names could be written by their sections. It is not just for experts to contribute, but also students who happened to be doing their homework. The student's contributions can be done by adding a question and perhaps the solution. Thus, this method is expected to accelerate the creation of

[4] see also in Franks, Nigel R.; "Army Ants: A Collective Intelligence," American Scientist, 77:139, 1989 (see for information `http://www.ex.ac.uk/bugclub/raiders.html`)

these high quality books.

These books are written in a similar manner to the open source software process. Someone has to write the skeleton and hopefully others will add "flesh and skin." In this process, chapters or sections can be added after the skeleton has been written. It is also hoped that others will contribute to the question and answer sections in the book. But more than that, other books contain data[5] which can be typeset in LaTeX. These data (tables, graphs and etc.) can be redone by anyone who has the time to do it. Thus, the contributions to books can be done by many who are not experts. Additionally, contributions can be made from any part of the world by those who wish to translate the book.

It is hoped that the books will be error-free. Nevertheless, some errors are possible and expected. Even if not complete, better discussions or better explanations are all welcome to these books. These books are intended to be "continuous" in the sense that there will be someone who will maintain and improve the books with time (the organizer(s)).

These books should be considered more as a project than to fit the traditional definition of "plain" books. Thus, the traditional role of author will be replaced by an organizer who will be the one to compile the book. The organizer of the book in some instances will be the main author of the work, while in other cases only the gate keeper. This may merely be the person who decides what will go into the book and what will not (gate keeper). Unlike a regular book, these works will have a version number because they are alive and continuously evolving.

What Has been So Far

The undersigned of this document intends to be the organizer–author–coordinator of the projects in the following areas:

Table -1. Books under development in Potto project.

Project Name	Progress	Remarks	Version	Availability for Public Download	Number DownLoads
Compressible Flow	beta		0.4.8.4	✔	120,000
Die Casting	alpha		0.1	✔	60,000
Dynamics	NSY		0.0.0	✖	-
Fluid Mechanics	alpha		0.1.8	✔	15,000
Heat Transfer	NSY	Based on Eckert	0.0.0	✖	-

[5] Data are not copyrighted.

Table -1. Books under development in Potto project. (continue)

Project Name	Progress	Remarks	Version	Availability for Public Download	Number DownLoads
Mechanics	NSY		0.0.0	✖	-
Open Channel Flow	NSY		0.0.0	✖	-
Statics	early alpha	first chapter	0.0.1	✖	-
Strength of Material	NSY		0.0.0	✖	-
Thermodynamics	early alpha		0.0.01	✖	-
Two/Multi phases flow	NSY	Tel-Aviv'notes	0.0.0	✖	-

NSY = Not Started Yet

The meaning of the progress is as:

- The Alpha Stage is when some of the chapters are already in a rough draft;
- in Beta Stage is when all or almost all of the chapters have been written and are at least in a draft stage;
- in Gamma Stage is when all the chapters are written and some of the chapters are in a mature form; and
- the Advanced Stage is when all of the basic material is written and all that is left are aspects that are active, advanced topics, and special cases.

The mature stage of a chapter is when all or nearly all the sections are in a mature stage and have a mature bibliography as well as numerous examples for every section. The mature stage of a section is when all of the topics in the section are written, and all of the examples and data (tables, figures, etc.) are already presented. While some terms are defined in a relatively clear fashion, other definitions give merely a hint on the status. But such a thing is hard to define and should be enough for this stage.

The idea that a book can be created as a project has mushroomed from the open source software concept, but it has roots in the way science progresses. However, traditionally books have been improved by the same author(s), a process in which books have a new version every a few years. There are book(s) that have continued after their author passed away, i.e., the *Boundary Layer Theory* originated[6] by Hermann Schlichting but continues to this day. However, projects such as the Linux Documentation project

[6]Originally authored by Dr. Schlichting, who passed way some years ago. A new version is created every several years.

demonstrated that books can be written as the cooperative effort of many individuals, many of whom volunteered to help.

Writing a textbook is comprised of many aspects, which include the actual writing of the text, writing examples, creating diagrams and figures, and writing the LaTeX macros[7] which will put the text into an attractive format. These chores can be done independently from each other and by more than one individual. Again, because of the open nature of this project, pieces of material and data can be used by different books.

[7]One can only expect that open source and readable format will be used for this project. But more than that, only LaTeX, and perhaps troff, have the ability to produce the quality that one expects for these writings. The text processes, especially LaTeX, are the only ones which have a cross platform ability to produce macros and a uniform feel and quality. Word processors, such as OpenOffice, Abiword, and Microsoft Word software, are not appropriate for these projects. Further, any text that is produced by Microsoft and kept in "Microsoft" format are against the spirit of this project In that they force spending money on Microsoft software.

Prologue For This Book

Version 0.1 January 12, 2009

pages 213 size 1.5M

Die casting was my focus of my Ph.D. thesis which admittedly, is not my preferred choice. Dr. Eckert, my adviser, asked me to work on die casting and that is where I developed my knowledge. The first thing that I have done is a literature review which force me to realize that that there is very little scientific known about how to design the die casting process. I have reviewed works/papers by from of Ohio State University by A. Miller, Brevick, J. Wallace from Case Western, Murry from Australia etc. Scientists are categorized in the following categories, Free thinkers, Cathedral builder, research managers, dust collectors (important work but minor), and thus who should be in science and thus those who are very lucky. This author feel that he, in same sense, very luck that die casting research is infested with thus who should be in science.

Like moving from the stone age to modern time, this author is using this book as a tool in his attempt to convert die casting design process to be based on real scientific principles. I have found that the book early version (0.0.3) of the have been downloaded over 50,000. It is strange to me that the fact that many were using the economical part of the book to explain many other the economical problems of large scale manufacturing processes. As I am drifting towards a different field (renewal energy), I still have interest in this material but with different aspects will be emphasized. Subjects like Fanno Flow that was as written as appendix will be expanded. Moreover, material like the moving shock issue will be explained and add to process description was omitted in the previous version. While this topic is not directly affecting die casting, the issue of future value will be discussed.

Version 0.0.3 October 9, 1999

pages 178 size 3.2M

This book is the first and initial book in the series of POTTO project books. This book started as a series of articles to answer both specific questions that I have been asked, as well as questions that I was curious about myself. While addressing these questions, I realized that many commonly held "truths" about die-casting were scientifically incorrect. Because of the importance of these results, I have decided to make them available to the wider community of die-casting engineers. However, there is a powerful group of individuals who want to keep their monopoly over "knowledge" in the die-casting industry and to prevent the spread of this information.[8] Because of this, I have decided that the best way to disseminate this information is to write a book. This book is written in the spirit of my adviser and mentor E.R.G. Eckert. Eckert, aside from his research activity, wrote the book that brought a revolution in the education of the heat transfer. Up to Eckert's book, the study of heat transfer was without any dimensional analysis. He wrote his book because he realized that the dimensional analysis utilized by him and his adviser (for the post doc), Ernst Schmidt, and their colleagues, must be taught in engineering classes. His book met strong criticism in which some called to "burn" his book. Today, however, there is no known place in world that does not teach according to Eckert's doctrine. It is assumed that the same kind of individual(s) who criticized Eckert's work will criticize this work. As a wise person says "don't tell me that it is wrong, show me what is wrong"; this is the only reply. With all the above, it must be emphasized that this book is not expected to revolutionize the field but change some of the way things are taught.

The approach adapted in this book is practical, and more hands–on approach. This statement really meant that the book is intent to be used by students to solve their exams and also used by practitioners when they search for solutions for practical problems. So, issue of proofs so and so are here only either to explain a point or have a solution of exams. Otherwise, this book avoids this kind of issues.

This book is divided into two parts. The first discusses the basic science required by a die–casting engineer; the second is dedicated to die-casting–specific science. The die-casting specific is divided into several chapters. Each chapter is divided into three sections: section 1 describes the "commonly" believed models; section 2 discusses why this model is wrong or unreasonable; and section 3 shows the correct, or better, way to do the calculations. I have made great efforts to show what existed before science "came" to die casting. I have done this to show the errors in previous models which make them invalid, and to "prove" the validity of science. I hope that, in the second edition, none of this will be needed since science will be accepted and will have gained validity in the die casting community. Please read about my battle to get the information out and how the establishment react to it.

---

[8] Please read my correspondence with NADCA editor Paul Bralower and Steve Udvardy. Also, please read the references and my comments on pQ^2 .

Plea for LaTeX usage

Is it only an accident that both the quality of the typesetting of papers in die casting congress and their technical content quality is so low? I believe there is a connection. All the major magazines of the the scientific world using TeX or LaTeX, why? Because it is very easy to use and transfer (via the Internet) and, more importantly, because it produces high quality documents. NADCA continued to produce text on a low quality word processor. Look for yourself; every transaction is ugly.

Linux has liberated the world from the occupation and the control of Microsoft OS. We hope to liberate the NADCA Transaction from such a poor quality word processor. TeX and all (the good ones) supporting programs are free and available every where on the web. There is no reason not to do it. Please join me in improving NADCA's Transaction by supporting the use of LaTeX by NADCA.

Will I Be in Trouble?

Initial part

Many people have said I will be in trouble because I am telling the truth. Those with a vested interest in the status quo (North American Die Casting Association, and thus research that this author exposed there poor and or erroneous work). will try to use their power to destroy me. In response, I challenge my opponents to show that they are right. If they can do that, I will stand wherever they want and say that I am wrong and they are right. However, if they cannot prove their models and practices are based on solid scientific principles, nor find errors with my models (and I do not mean typos and English mistakes), then they should accept my results and help the die–casting industry prosper.

People have also suggested that I get life insurance and/or good lawyer because my opponents are very serious and mean business; the careers of several individuals are in jeopardy because of the truths I have exposed. If something does happen to me, then you, the reader, should punish them by supporting science and engineering and promoting the die–casting industry. By doing so, you prevent them from manipulating the industry and gaining additional wealth.

For the sake of my family, I have, in fact, taken out a life insurance policy. If something does happen to me, please send a thank you and work well done card to my family.

The Continued Struggle

It was exposed that second reviewer that appear in this book is Brevick from Ohio. It strange that in a different correspondence he say that he cannot wait to get this author futur work. This part is holding for some juicy details.

How This Book Was Written

This book started because I was frustrated with the system that promote erroneous research. Then, I realized that the book cannot be "stolen" if it under open content. The die casting process is interesting enough to insert my contributions. I have found that works or model in this area are lack of serious scientific principles. I have started to write class notes to my clients and I add my research work to create this book. During the writing I add the material on economy which I felt is missing piece of knowledge in the die casting engineering world.

Of course, this book was written on Linux (Micro$oftLess book). This book was written using the vim editor for editing (sorry never was able to be comfortable with emacs). The graphics were done by TGIF, the best graphic program that this author experienced so far. The figures were done by grap but will be modified to gle. The spell checking was done by ispell, and hope to find a way to use gaspell, a program that currently cannot be used on new Linux systems. The figure in cover page was created by Genick Bar-Meir, and is copylefted by him.

Abstract

Die-casting engineers have to compete not only with other die-casting companies, but also against other industries such as plastics, and composite materials. Clearly, the "black art" approach, which has been an inseparable part of the engineer's tools, is in need of being replaced by a scientific approach. Excuses that "science has not and never will work" need to be replaced with "science does work". All technologies developed in recent years are described in a clear, simple manner in this book. All the errors of the old models and the violations of physical laws are shown. For example, the "common" pQ^2 diagram violates many physical laws, such as the first and second laws of thermodynamics. Furthermore, the "common" pQ^2 diagram produces trends that are the opposite of reality, which are described in this book.

The die casting engineer's job is to produce maximum profits for the company. In order to achieve this aim, the engineer must design high quality products at a minimum cost. Thus, understanding the economics of the die casting design and process are essential. These are described in mathematical form for the first time in this volume. Many new concepts and ideas are also introduced. For instance, how to minimize the scrap/cost due to the runner system, and what size of die casting machine is appropriate for a specific project.

The die-casting industry is undergoing a revolution, and this book is part of it. One reason (if one reason can describe the situation) companies such as Doehler Jorvis (the biggest die caster in the world) and Shelby are going bankrupt is that they do not know how to calculate and reduce their production costs. It is my hope that die-casters will turn such situations around by using the technologies presented in this book. I believe this is the only way to keep the die casting professionals and the industry itself, from being "left in the dust."

Preface

"In the beginning, the POTTO project was without form, and void; and emptiness was upon the face of the bits and files. And the Fingers of the Author moved upon the face of the keyboard. And the Author said, Let there be words, and there were words."[9].

This book, "Fundamentals of Die Casting Design," describes the fundamentals of die casting process design and economics for engineers and others. This book is designed to fill the gap and the missing book on economy and scientific principles of die casting. It is hoped that the book could be used as a reference book for people who have at least some basics knowledge of science areas such as calculus, physics, etc. It has to realized the some material is very advance and required knowledge of fluid mechanics particularly compressible flow and open channel flow. This author's popular book on compressible flow should provide the introductory in that area. The readers' reactions to this book and the usage of the book as a textbook suggested that the chapter which deals with economy should be expand. In the following versions this area will strength and expended.

The structure of this book is such that many of the chapters could be usable independently. For example, if you need information about, say, economy of the large scale productions, you can read just chapter (12). I hope this makes the book easier to use as a reference manual. However, this manuscript is first and foremost a textbook, and secondly a reference manual only as a lucky coincidence.

I have tried to describe why the theories are the way they are, rather than just listing "seven easy steps" for each task. This means that a lot of information is presented which is not necessary for everyone. These explanations have been marked as such and can be skipped. Reading everything will, naturally, increase your understanding of the many aspects of fluid mechanics.

[9]To the power and glory of the mighty God. This book is only to explain his power.

This book is written and maintained on a volunteer basis. Like all volunteer work, there is a limit on how much effort I was able to put into the book and its organization. Moreover, due to the fact that English is my third language and time limitations, the explanations are not as good as if I had a few years to perfect them. Nevertheless, I believe professionals working in many engineering fields will benefit from this information. This book contains many worked examples, which can be very useful for many.

I have left some issues which have unsatisfactory explanations in the book, marked with a Mata mark. I hope to improve or to add to these areas in the near future. Furthermore, I hope that many others will participate of this project and will contribute to this book (even small contributions such as providing examples or editing mistakes are needed).

I have tried to make this text of the highest quality possible and am interested in your comments and ideas on how to make it better. Incorrect language, errors, ideas for new areas to cover, rewritten sections, more fundamental material, more mathematics (or less mathematics); I am interested in it all. I am particularly interested in the best arrangement of the book. If you want to be involved in the editing, graphic design, or proofreading, please drop me a line. You may contact me via Email at barmeir at gmail dot com.

Naturally, this book contains material that never was published before (sorry cannot avoid it). This material never went through a close content review. While close content peer review and publication in a professional publication is excellent idea in theory. In practice, this process leaves a large room to blockage of novel ideas and plagiarism. For example, Brevick from Ohio State is one the individual who attempt to block this author idea on pQ^2 diagram. If you would like to critic to my new ideas please send me your comment(s). However, please do not hide your identity, it will cloud your motives.

Several people have helped me with this book, directly or indirectly. I would like to especially thank to my adviser, Dr. E. R. G. Eckert, whose work was the inspiration for this book. I also would like to thank to Jannie McRotien (Open Channel Flow chapter) and Tousher Yang for their advices, ideas, and assistance.

I encourage anyone with a penchant for writing, editing, graphic ability, LaTeX knowledge, and material knowledge and a desire to provide open content textbooks and to improve them to join me in this project. If you have Internet e-mail access, you can contact me at “barmeir@gmail.com”.

CHAPTER 1

Introduction

In the recent years, many die casting companies have gone bankrupt (Doehler–Jarvis and Shelby to name a few) and many other die casting companies have been sold (St. Paul Metalcraft, Tool Products, OMC etc.). What is/are the reason/s for this situation? Some blame poor management. Others blame bad customers (which is mostly the automobile industry). Perhaps there is something to these claims. Nevertheless, one can see that the underlying reasons are the missing knowledge of how to calculate if there are profits for a production line and how to design, so that costs will be minimized. To demonstrate how the absurd situation is the fact that there is not even one company today that can calculate the actual price of any product that they are producing. Moreover, if a company is able to produce a specific product, no one in that company looks at the redesign (mold or process) in order to reduce the cost systematically.

In order to compete with other industries and other companies, the die casting industry **must** reduce the cost as much as possible (20% to 40%) and lead time significantly (by 1/2 or more). To achieve these goals, the engineer must learn to connect mold design to the cost of production (charged to the customer) and to use the correct scientific principals involved in the die casting process to reduce/eliminate the guess work. This book is part of the revolution in die casting by which science is replacing the black art of design. For the first time, a link between the cost and the design is spelled out. Many new concepts, based on scientific principles, are introduced. The old models, which was plagued by the die casting industry for many decades, are analyzed, their errors are explained and the old models are superseded.

"Science is good, but it is not useful in the floor of our plant!!" George Reed, the former president of SDCE, in 1999 announced in a meeting in the local chapter (16) of NADCA. He does not believe that there is A relationship between "science" and what he does with the die casting machine. He said that because he does not follow NADCA recommendations, he achieves good castings. For instance, he stated that the common and NADCA supported, recommendation in order to increase the gate velocity, plunger diameter needs to be decreased. He said that because he does not follow this recommendation, and/or others, that is the reason his succeeds in obtaining good castings. He is right and wrong. He is right not to follow the NADCA recommendations since they violate many basic scientific principles. One should expect that models violating scientific principles would produce unrealistic results. When such results occur, this should actually strengthen the idea that science has validity. The fact that models which appear in books today are violating scientific principals and therefore do not work should actually convince him, and others, that science does have validity. Mr. Reed is right (in certain ranges) to increase the diameter in order to increase the gate velocity as will be covered in Chapter 7.

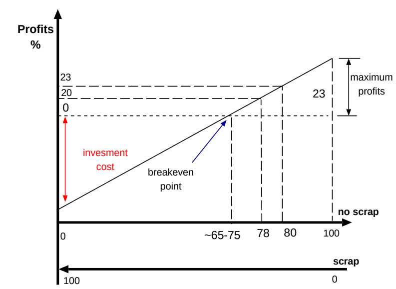


Fig. -1.1. The profits as a function of the amount of the scrap.

The above example is but one of many of models that are errant and in need of correction. To this date, the author has not found so much as a single "commonly" used model that has been correct in its conclusions, trends, and/or assumptions. The wrong models/methods that have plagued the industry are: 1) critical slow plunger velocity, 2) pQ^2 diagram, 3) plunger diameter calculations, 4) runner system design, 5) vent system design, etc These incorrect models are the reasons that "science" does not work. The models presented in this book are here for the purpose of answering the questions of design in a scientific manner which will result in reduction of costs and increased product quality.

Once the reasons to why "science" does not work are clear, one should learn the correct models for improving quality, reducing lead time and reducing production cost. The main underlying reason people are in the die casting business is to make money. One has to use science to examine what the components of production cost/scrap are and how to minimize or eliminate each of them to increase profitability. The underlying purpose of this book is to help the die caster to achieve this target.

1.1 The Importance of Reducing Production Costs

Contrary to popular belief, a reduction of a few percentage points of the production cost/scrap does not translate into the same percentage of increase in profits. The increase is a little bit more complicated function. To study the relationship further, see Figure 1.1 where profits are plotted as a function of the scrap. A linear function describes the relationship, when the secondary operations are neglected. The maximum loss occurs when all the material turned out to be scrap and it is referred to as the "investment cost". On the other hand, maximum profits occur when all the material becomes products (no scrap of any kind (see Figure 1.1). The breakeven point (BEP) has to exist somewhere between these two extremes. Typically, for the die casting industry, the breakeven point lies within the range of 55%–75% product (or 25%-35% scrap). Typical profits in the die casting industry are or should be about 20%. When the profits fails below 15% or typical profit in the stock exchange then the production should stop. From Figure 1.1 it can be noticed that

$$\text{relative change in profits\%} = \left(\frac{\text{new product percent} - \text{BEP}}{\text{old product percent} - \text{BEP}} - 1 \right) \times 100 \qquad (1.1)$$

Example 1.1:

What would be the effect on the profits of a small change (2%) in a amount of scrap for a job with 22% scrap (78% product) and with breakeven point of 65%?

SOLUTION

$$\left(\frac{80 - 65}{78 - 65} - 1 \right) \times 100 = 15.3\%$$

A reduction of 2% in a amount of the scrap to be 20% (80% product) results in increase of more than 15.3% in the profits.

End Solution

This is a very substantial difference. Therefore, a much bigger reduction in scrap will result in much, much bigger profits.

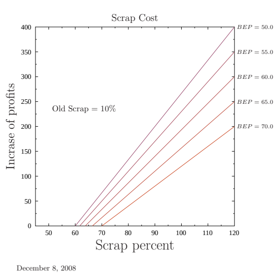


Fig a. *For BEP= 10%*

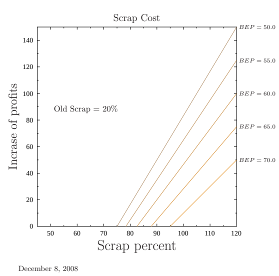


Fig b. *For BEP= 10%*

Fig. -1.2. The left graph depicts the increase of profits as reduction of the scrap for 10% + BEP. The right graph depicts same for for 20% + BEP.

To analysis this point further Figure intro:fig:scrapCostBEP is built for two "old" scrap values, 10% more than the BEP on the left and 20% more than BEP on the right. The two figures (left and right) in Figure intro:fig:scrapCostBEP demonstrate that The higher BEP the change in reduction of scrap is more important. The lower the old scrap point is the more important the reduction is.

1.2 Designed/Undesigned Scrap/Cost

There can be many definitions of scrap. The best definition suited to the die casting industry should be defined as all the metal that did not become a product. There are two kinds of scrap/cost: 1) those that can be eliminated, and 2) those that can only be minimized. The first kind is referred to here as the undesigned scrap and the second is referred as designed scrap. What is the difference? It is desired not to have rejection of any part (the rejection should be zero) and of course it is not designed. Therefore, this is the undesigned scrap/cost. However, it is impossible to eliminate the runner completely and it is desirable to minimize its size in such a way that the cost will be minimized. This minimization of cost and this minimum scrap is the designed scrap/cost. The die casting engineer must distinguish between these two scrap components in order to be able to determine what should be done and what cannot be done.

Science can make a significant difference; for example, it is possible to calculate the critical slow plunger velocity and thereby eliminating (almost) air entrainment in the shot sleeve in order to minimize the air porosity. This means that air porosity will be reduced and marginal products (even poor products in some cases) are converted into good quality products. In this way, the undesigned scrap can be eliminated or minimized. Additional way of minimizing the scrap is changing several parameters. The minimum scrap/cost can be achieved when a combination of the smallest runner volume and the cheapest die casting machine are selected for a single cavity. Similar analysis can be done for multiply cavity molds. This topic will be studied further in Chapter 12.

The possibility that a parameter, which reduces the designed scrap/cost will, at the same time, reduce the undesigned scrap/cost. An example of such a parameter is the venting system design. It will be shown that there is a critical design below which air/gas is exhausted easily and above which air is trapped. In the later case, the air/gas pressure builds up and results in a poor casting (large amount of porosity) The meaning of the critical design and above and below critical design will be presented in Chapter 9. The analysis of the vent system demonstrates that a design much above the critical design and design just above the critical design yielding has almost the same results– small amount of air entrainment. One can design the vent just above the critical design so the design scrap/cost is reduced to a minimum amount possible. Now both targets have been achieved: less rejections (undesigned scrap) and less vent system volume (designed scrap). It also possible to have an opposite case in which reduction of designed scrap results in poor design. The engineer has to be aware of these points.

1.3 Linking the Production Cost to the Product Design

It is sound accounting practice to tie the cost of every aspect of production to the cost to be charged to the customer. Unfortunately, the practice today is such that the price of the products are determined by some kind of average based on the part weight plus geometry and not on the actual design and production costs. Furthermore, this idea is also perpetuated by researchers who do not have any design factor [14]. Here it is advocated to price according to the actual design and production costs. It is believed that better pricing results from such a practice. In today's practice, even after the project is finished, no one calculates the actual cost of production, let alone calculating the actual profits. The consequences of such a practice are clear: it results in no push for better design and with no idea which jobs make profits and which do not. Furthermore, considerable financial cost is incurred which could easily be eliminated. Several chapters in this book are dedicated to linking the design to the cost (end-price).

1.4 Historical Background

Die casting is, relatively speaking a very forgiving process, in which after tinkering with the several variables one can obtain a medium quality casting. For this reason there has not been any real push toward doing good research. Hence, all the major advances in the understanding of the die casting process were not sponsored by any of die casting institutes/associations. Many of the people in important positions in the die casting industry suffer from what is known as the "Detroit attitude", which is very difficult to change. "*We are making a lot of money so why change? and if do not the Government will pay for it.*". Moreover, the controlling personnel on the research funds believe that the die casting is a metallurgical manufacturing process and therefore, the research has to be carried out by either Metallurgical Engineers or Industrial Engineers. Furthermore, should come as no surprise – that people–in–charge of the research funding fund their own research. One cannot wonder if there is a relationship between so many erroneous models which have been produced and the personnel controlling the research funding. A highlight of the major points of the progress of the understanding is described herein.

The vent system design requirements were studied by some researchers, for example Suchs, Veinik, and Draper and others. These models, however, are unrealistic and do not provide no relation to the physics or realistic picture of the real requirements or of the physical situation since they ignore the major point, the air compressibility. However this research extremely poor, it highlights the idea that venting design is a must.

One of the secrets of the black art of design was that there is a range of gate velocity which creates good castings depending on the alloy properties being casted. The existence of a minimum velocity hints that a significant change in the liquid metal flow pattern occurs. Veinik linked the gate velocity to the flow pattern (atomization) and provide a qualitative physical explanation for this occurrence. Experimental work [25] showed that liquid metals, like other liquids, flow in three main patterns: a continuous flow jet, a coarse particle jet, and an atomized particle jet. Other researchers utilized the water analogy method to study flow inside the cavity for example, [6]. At present, the (minimum) required gate velocity is supported by experimental evidence which is related to the flow patterns. However, the numerical value is unknown because the experiments were poorly conducted for example, [30] the differential equations that have been "solved" are not typical to die casting. Discussion about this poor research is presented in Chapter 3. At this stage, this question is not understood.

In the late 70's, an Australian group [12] suggested adopting the pQ^2 diagram for die casting in order to calculate the gate velocity, the gate area, and other parameters.

As with all the previous models they missed the major points of the calculations. As will be shown in Chapter 7, the Australian's model produce incorrect results and predict trends opposite to reality. This model took root in die casting industry for the last 25 years. Yet, one can only wonder why this well established method (the supply and demand theory which was build by Fanno (the brother of other famous Fanno from Fanno flow), which was introduced into fluid mechanics in the early of this century, reached the die casting only in the late 70's and was then erroneously implemented. This methods now properly build for the first time for the die casting industry in this book.

Until the 1980's there was no model that assisted the understanding air entrapment in the shot sleeve. Garber described the hydraulic jump in the shot sleeve and called it the "wave", probably because he was not familiar with this research area. He also developed the erroneous model which took root in the industry in spite the fact that **it never works**. One can only wonder why any die casting institutes/associations have not published this fact. Moreover, NADCA and other institutes continue to funnel large sums of money to the researchers (for example, Brevick from Ohio State) who used Garber's model even after they knew that Garber's model was totally wrong.

The turning point of the understanding was when Prof. Eckert, the father of modern heat transfer, introduced the dimensional analysis applied to the die casting process. This established a scientific approach which provided an uniform schemata for uniting experimental work with the actual situations in the die casting process. Dimensional analysis demonstrates that the fluid mechanics processes, such as filling of the cavity with liquid metal and evacuation/extraction of the air from the mold, can be dealt when the heat transfer is assumed to be negligible. However, the fluid mechanics has to be taken into account in the calculations of the heat transfer process (the solidification process).

This proved an excellent opportunity for "simple" models to predict the many parameters in the die casting process, which will be discussed later in this book. Here, two examples of new ideas that mushroomed in the inspiration of prof. Eckert's work. It has been shown that [5] the net effect of the reactions is negligible. This fact is contradictory to what was believed at that stage. The development of the critical vent area concept provided the major guidance for 1) the designs to the venting system, and 2) criterion when the vacuum system needs to be used. In this book, many of the new concepts and models, such as economy of the runner design, plunger diameter calculations, minimum runner design, etc, are described for the first time.

1.5 Numerical Simulations

Numerical simulations have been found to be very useful in many areas which lead many researchers attempting to implement them into die casting process. Considerable research work has been carried out on the problem of solidification including fluid flow which is known also as Stefan problems [21]. Minaie et al in one of the pioneered work use this knowledge and simulated the filling and the solidification of the cavity using finite difference method. Hu et al used the finite element method to improve the grid problem and to account for atomization of the liquid metal. The atomization model in the last model was based on the mass transfer coefficient. This model atomization is not appropriate. Clearly, this model is in waiting to be replaced by a realistic model to describe the mass transfer[1]. The Enthalpy method was further exploded by Swaminathan and Voller and others to study the filling and solidification problem.

While numerical simulation looks very promising, all the methods (finite difference, finite elements, or boundary elements etc) [2] suffer from several major drawbacks which prevents them from yielding reasonable results.

- There is no theory (model) that explains the heat transfer between the mold walls and the liquid metal. The lubricant sprayed on the mold change the characteristic of the heat transfer. The difference in the density between the liquid phase and solid phase creates a gap during the solidification process between the mold and the ingate which depends on the geometry. For example, Osborne et al showed that a commercial software (MAGMA) required fiddling with the heat transfer coefficient to get the numerical simulation match the experimental results[3].

- As it was mentioned earlier, it is not clear when the liquid metal flows as a spray and when it flows as continuous liquid. Experimental work has demonstrated that the flow, for a large part of the filling time, is atomized [4].

- The pressure in the mold cavity in all the commercial codes are calculated without taking into account the resistance to the air flow out. Thus, built–up pressure in the cavity is poorly estimated, or even not realistic, and therefore the characteristic flow of the liquid metal in the mold cavity is poorly estimated as well.

- The flow in all the simulations is assumed to be turbulent flow. However, time and space are required to achieved a fully turbulent flow. For example, if the flow at the entrance to a pipe with the typical conditions in die casting is laminar (actually it is a plug flow) it will take a runner with a length of about 10[m] to achieved fully developed flow. With this in mind, clearly some part of the flow is laminar. Additionally, the solidification process is faster compared to the dissipation process in the initial stage, so it is also a factor in changing the flow from a turbulent (in case the flow is turbulent) to a laminar flow.

- The liquid metal velocity at the entrance to the runner is assumed for the numerical simulation and not calculated. In reality this velocity has to be calculated utilizing the pQ^2 diagram.

- If turbulence exists in the flow field, what is the model that describes it adequately? Clearly, model such $k-\epsilon$ are based on isentropic homogeneous with mild change in the properties cannot describe situations where the flow changes into two-phase flow (solid-liquid flow) etc.

---

[1] One finds that it is the easiest to critic one's own work or where he/she was involved.

[2] Commercial or academic versions.

[3] Actually, they attempted to prove that the software is working very well. However, the fact that coefficient need to field is excellent proof why this work is meaningless.

- The heat extracted from the die is done by cooling liquid (oil or water). In most models (all the commercial models) the mechanism is assumed to be by "regular cooling". In actuality, some part of the heat is removed by boiling heat transfer.

- The governing equations in all the numerical models, that I am aware of, neglect the dissipation term in during the solidification. The dissipation term is the most important term in that case.

One wonders how, with unknown flow pattern (or correct flow pattern), unrealistic pressure in the mold, wrong heat removal mechanism (cooling method), erroneous governing equation in the solidification phase, and inappropriate heat transfer coefficient, a simulation could produce any realistic results. Clearly, much work is need to be done in these areas before any realistic results should be expected from any numerical simulation. Furthermore, to demonstrate this point, there are numerical studies that assume that the flow is turbulent, continuous, no air exist (or no air leaving the cavity) and proves with their experiments that their model simulate "reality" [23]. On the other hand, other numerical studies assumed that the flow does not have any effect on the solidification and of course have their experiments to support this claim [11]. Clearly, this contradiction suggest several options:

- Both of the them are right and the model itself does not matter.

- One is right and the other one is wrong.

- Both of them are wrong.

The third research we mentioned here is an example where the calculations can be shown to be totally wrong and yet the researchers have experimental proofs to back them up. Viswanathan et al studied a noble process in which the liquid metal is poured into the cavity and direct pressure is applied to the cavity. In their calculations the authors assumed that metal enter to the cavity and fill the whole entrance (gate) to the cavity. Based on this assumption their model predict defects in certain geometry. A critical examination of this model present the following. The assumption of no air flow out by the authors (was "explained" privately that air amount is a small and therefore not important) is very critical as will be shown here. The volumetric air flow rate into the cavity has to be on average equal to liquid metal flow rate (conservation of volume for constant density). Hence, air velocity has to be approximately infinite to achieve zero vent area. Conversely, if the assumption that the air flows in the same velocity as the liquid entering the cavity, liquid metal flow area is a half what is assume in the researchers model. In realty, the flow of the liquid metal is in the two phase region and in this case, it is like turning a bottle full of water over and liquid inside flows as "blobs" [4]. More information can be found on reversible flow in this author book in Potto series of "Basics of Fluid Mechanics." In this case the whole calculations do not have much to do with reality since the velocity is not continuous and different from what was calculated.

[4] Try it your self! fill a bottle and turn it upside and see what happens.

Another example of such study is the model of the flow in the shot sleeve by Backer and Sant from EKK [2][5]. The researchers assumed that the flow is turbulent and they justified it because they calculated and found a "jet" with extreme velocity. Unfortunately, all the experimental evidence demonstrate that there is no such jet [24]. It seems that this jet results from the "poor" boundary and initial conditions[6]. In the presentation, the researchers also stated that results they obtained for laminar and turbulent flow were the same[7] while a simple analysis can demonstrate the difference is very large. Also, one can wonder how liquid with zero velocity to be turbulent. With these results one can wonder if the code is of any value or the implementation is at fault.

The bizarre belief that the numerical simulations are a panacea to all the design problem is very popular in the die casting industry. Any model has to describe and account for the physical situation in order to be useful. Experimental evidence which is supporting wrong models as a real evidence is nonsense. Clearly some wrong must be there. For example, see the paper by Murray and colleague in which they use the fact that two unknown companies (somewhere in the outer space maybe?) were using their model to claim that it is correct.. A proper way can be done by numerical calculations based on real physics principles which produce realistic results. Until that point come, the reader should be suspicious about any numerical model and its supporting evidence.[8]

1.6 "Integral" Models

Unfortunately, the numerical simulations of the liquid metal flow and solidification process do not yield reasonable results at the present time. This problem has left the die casting engineers with the usage of the "integral approach" method. In this method the calculations are broken into simplified models. One of the most important tool in this approach is the pQ^2 diagram, one of the manifestations of the supply and demand theory. In this diagram, an engineer insures that die casting machine ability can fulfill the die mold design requirements; the liquid metal is injected at the right velocity range and the filling time is small enough to prevent premature freezing. One can, with the help of the pQ^2 diagram and by utilizing experimental values for desired filling time and gate velocities improve the quality of the casting. The gate velocity has to be above a certain value to assure atomization and below a critical value to prevent erosion of the mold. This two values are experimental and no reliable theory is available today known. The correct model for the pQ^2 diagram has been developed and will be presented in Chapter 7. A by–product of the above model is the plunger diameter calculations and it is discussed in Chapter 7.

---

[5]It was suggested by several people that the paper was commissioned by NADCA to counter Bar-Meir's equation to shot sleeve. This fact is up to the reader to decide if it is correct.

[6]The boundary and initial conditions were not spelled out in the paper!! However they were implicitly stated in the presentation.

[7]So why to use the complicate turbulent model?

[8]With all these harsh words, I would like to take the opportunity for the record, I do think that work by Davey's group is a good one. They have inserted more physics (for example the boiling heat transfer) into their models which I hope in the future, leads us to have realistic numerical models.

It turned out that many of the design parameters in die casting have a critical point above which good castings are produced and below which poor castings are produced. Furthermore, **much** above and **just** above the critical point do not change much the quality but costs much more. This fact is where the economical concepts plays a significant role. Using these concepts, one can increase the profitability significantly, and obtain very good quality casting and reduce the leading time. Additionally, the main cost components like machine cost and other are analyzed which have to be taken into considerations when one chooses to design the process will be discussed in the Chapter 12.

Porosity can be divided into two main categories; shrinkage porosity and gas/air entrainment. The porosity due to entrapped gases constitutes a large part of the total porosity. The creation of gas/air entrainment can be attributed to at least four categories: lubricant evaporation (and reaction processes[9]), vent locations (last place to be filled), mixing processes, and vent/gate area. The effects of lubricant evaporation have been found to be insignificant. The vent location(s) can be considered partially solved since only qualitative explanation exist. The mixing mechanisms are divided into two zones: the mold, and the shot sleeve. Some mixing processes have been investigated and can be considered solved. The requirement on the vent/gate areas is discussed in Chapter 9. When the mixing processes are very significant in the mold, other methods are used and they include: evacuating the cavities (vacuum venting), Pore Free Technique (in zinc and aluminum casting) and squeeze casting. The first two techniques are used to extract the gases/air from the shot sleeve and die cavity before the gases have the opportunity to mix with the liquid metal. The squeeze casting is used to increase the capillary forces and therefore, to minimize the mixing processes. All these solutions are cumbersome and more expensive and should be avoided if possible.

The mixing processes in the runners, where the liquid metal flows vertically against gravity in relatively large conduit, are considered to be insignificant[10]. The enhanced air entrainment in the shot sleeve is attributed to operational conditions for which a blockage of the gate by a liquid metal wave occurs before the air is exhausted. Consequently, the residual air is forced to be mixed into the liquid metal in the shot sleeve. With Bar-Meir's formula, one can calculate the correct critical slow plunger velocity and this will be discussed in Chapter 8.

1.7 Summary

It is an exciting time in the die casing industry because for the first time, an engineer can start using real science in designing the runner/mold and the die casting process. Many new models have been build and many old techniques mistake have been removed. It is the new revolution in the die casting industry.

[9]Some researchers view the chemical reactions (e.g. release of nitrogen during solidification process) as category by itself.

[10]Some work has been carried out and hopefully will be published soon. And inside, in the book "Basic of Fluid Mechanics" in the two phase chapter some inside was developed.

CHAPTER 2

Basic Fluid Mechanics

2.1 Introduction

This chapter is presented to fill the void in basic fluid mechanics to the die casting community. It was observed that knowledge in this area cannot be avoided. The design of the process as well as the properties of casting (especially magnesium alloys) are determined by the fluid mechanics/heat transfer processes. It is hoped that others will join to spread this knowledge. There are numerous books for introductory fluid mechanics but the Potto series book "Basic of Fluid Mechanics" is a good place to start. This chapter is a summary of that book plus some pieces from the "Fundamentals of Compressible Flow Mechanics." It is hoped that the reader will find this chapter interesting and will further continue expanding his knowledge by reading the full Potto books on fluid mechanics and compressible flow.

First we will introduce the nature of fluids and basic concepts from thermodynamics. Later the integral analysis will be discussed in which it will be divided into introduction of the control volume concept and Continuity equations. The energy equation will be explained in the next section. Later, the momentum equation will be discussed. Lastly, the chapter will be dealing with the compressible flow gases. Here it will be refrained from dealing with topics such boundary layers, non–viscous flow, machinery flow etc which are not essential to understand the rest of this book. Nevertheless, they are important and it is advisable that the reader will read on these topics as well.

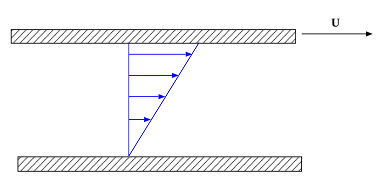


Fig. -2.1. The velocity distribution in Couette flow

2.2 What is fluid? Shear stress

Fluid, in this book, is considered as a substance that "moves" continuously and permanently when exposed to a shear stress. The liquid metals are an example of such substance. However, the liquid metals do not have to be in the liquidus phase to be considered liquid. Aluminum at approximately 400^0C is continuously deformed when shear stress are applied. The whole semi–solid die casting area deals with materials that "looks" solid but behaves as liquid.

2.2.1 What is Fluid?

The fluid is mainly divided into two categories: liquids and gases. The main difference between the liquids and gases state is that gas will occupy the whole volume while liquids has an almost fixed volume. This difference can be, for most practical purposes considered, sharp even though in reality this difference isn't sharp. The difference between a gas phase to a liquid phase above the critical point are practically minor. But below the critical point, the change of water pressure by 1000% only change the volume by less than 1 percent. For example, a change in the volume by more than 5% will require tens of thousands percent change of the pressure. So, if the change of pressure is significantly less than that, then the change of volume is at best 5%. Hence, the pressure will not affect the volume. In gaseous phase, any change in pressure directly affects the volume. The gas fills the volume and liquid cannot. Gas has no free interface/surface (since it does fill the entire volume).

2.2.2 What is Shear Stress?

The shear stress is part of the pressure tensor. However, here it will be treated as a separate issue. In solid mechanics, the shear stress is considered as the ratio of the force acting on area in the direction of the forces perpendicular to area. Different from solid, fluid cannot pull directly but through a solid surface. Consider liquid that undergoes a shear stress between a short distance of two plates as shown in Figure (2.2).

The upper plate velocity generally will be

$$U = f(A, F, h) \tag{2.1}$$

Where A is the area, the F denotes the force, h is the distance between the plates. From solid mechanics study, it was shown that when the force per area increases, the velocity of the plate increases also. Experiments show that the increase of height will increase the velocity up to a certain range. Consider moving the plate with a zero lubricant ($h \sim 0$) (results in large force) or a large amount of lubricant (smaller force). In this discussion, the aim is to develop differential equation, thus the small distance analysis is applicable.

For cases where the dependency is linear, the following can be written

$$U \propto \frac{h\,F}{A} \tag{2.2}$$

Equations (2.2) can be rearranged to be

$$\frac{U}{h} \propto \frac{F}{A} \tag{2.3}$$

Shear stress was defined as

$$\tau_{xy} = \frac{F}{A} \tag{2.4}$$

From equations (2.3) and (2.4) it follows that ratio of the velocity to height is proportional to shear stress. Hence, applying the coefficient to obtain a new equality as

$$\tau_{xy} = \mu \, \frac{U}{h} \tag{2.5}$$

Where μ is called the absolute viscosity or dynamic viscosity.

In steady state, the distance the upper plate moves after small amount of time, δt is

$$d\ell = U \, \delta t \tag{2.6}$$

From figure (2.2) it can be noticed that for a small angle, the regular approximation provides

$$d\ell = U \, \delta t = \overbrace{h \, \delta\beta}^{geometry} \tag{2.7}$$

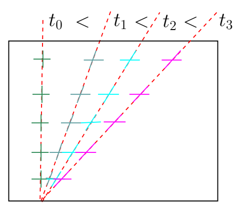


Fig. -2.2. The deformation of fluid due to shear stress as progression of time.

From equation (2.7) it follows that

$$U = h \frac{\delta\beta}{\delta t} \tag{2.8}$$

Combining equation (2.8) with equation (2.5) yields

$$\tau_{xy} = \mu \, \frac{\delta\beta}{\delta t} \tag{2.9}$$

If the velocity profile is linear between the plate (it will be shown later that it is consistent with derivations of velocity), then it can be written for small angle that

$$\frac{\delta\beta}{\delta t} = \frac{dU}{dy} \tag{2.10}$$

Materials which obey equation (2.9) are referred to as Newtonian fluid.

For liquid metal used in the die casting industry, this property should be considered as Newtonian fluid.

2.3 Thermodynamics and mechanics concepts

2.3.1 Thermodynamics

In this section, a review of several definitions of common thermodynamics terms is presented. This introduction is provided to bring familiarity of the material back to the student.

2.3.2 Basic Definitions

The following basic definitions are common to thermodynamics and will be used in this book.

Work

In mechanics, the work was defined as

$$\text{mechanical work} = \int \mathbf{F} \bullet \mathbf{d}\ell = \int P dV \tag{2.11}$$

This definition can be expanded to include two issues. The first issue that must be addressed, that work done on the surroundings by the system boundaries similarly is positive. Two, there is a transfer of energy so that its effect can cause work. It must be noted that electrical current is a work while heat transfer isn't.

System

This term will be used in this book and it is defined as a continuous (at least partially) fixed quantity of matter (neglecting Einstein's law effects). For almost all engineering purposes this law is reduced to two separate laws: mass conservation and energy conservation. Our system can receive energy, work, etc as long as the mass remains constant the definition is not broken.

Thermodynamics First Law

This law refers to conservation of energy in a non accelerating system. Since all the systems can be calculated in a non accelerating system, the conservation is applied to all systems. The statement describing the law is the following:

$$Q_{12} - W_{12} = E_2 - E_1 \tag{2.12}$$

The system energy is a state property. From the first law it directly implies that for process without heat transfer (adiabatic process) the following is true

$$W_{12} = E_1 - E_2 \tag{2.13}$$

Interesting results of equation (2.13) is that the way the work is done and/or intermediate states are irrelevant to final results. The internal energy is the energy that depends on the other properties of the system. Example: for pure/homogeneous and simple gases it depends on two properties like temperature and pressure. The internal energy is denoted in this book as E_U and it will be treated as a state property.

The system potential energy is dependent upon the body force. A common body force is gravity. For such body force, the potential energy is mgz where g is the gravity force (acceleration), m is the mass and the z is the vertical height from a datum. The kinetic energy is

$$K.E. = \frac{mU^2}{2} \tag{2.14}$$

Thus the energy equation can be written as

$$\boxed{\frac{m{U_1}^2}{2} + \overbrace{m\,g\,z_1}^{B_f} + E_{U\,1} + Q = \frac{m{U_2}^2}{2} + \overbrace{m\,g\,z_2}^{B_f} + E_{U\,2} + W} \tag{2.15}$$

where B_f is a body force. For the unit mass of the system equation (2.15) is transformed into

$$\boxed{\frac{{U_1}^2}{2} + gz_1 + E_{u\,1} + q = \frac{{U_2}^2}{2} + gz_2 + E_{u\,2} + w} \tag{2.16}$$

where q is the energy per unit mass and w is the work per unit mass. The "new" internal energy, E_u, is the internal energy per unit mass.

Since the above equations are true between arbitrary points, choosing any point in time will make it correct. Thus, differentiating the energy equation with respect to time yields the rate of change energy equation. The rate of change of the energy transfer is

$$\frac{DQ}{Dt} = \dot{Q} \tag{2.17}$$

In the same manner, the work change rate transferred through the boundaries of the system is

$$\frac{DW}{Dt} = \dot{W} \tag{2.18}$$

Since the system is with a fixed mass, the rate energy equation is

$$\dot{Q} - \dot{W} = \frac{D\,E_U}{Dt} + mU\frac{DU}{Dt} + m\frac{D\,g\,z}{Dt} \tag{2.19}$$

For the case were the body force, $B_f = g$, is constant with time like in the case of gravity equation (2.19) reduced to

$$\boxed{\dot{Q} - \dot{W} = \frac{D\,E_U}{Dt} + mU\frac{DU}{Dt} + m\,g\frac{D\,z}{Dt}} \tag{2.20}$$

The time derivative operator, D/Dt is used instead of the common notation because it refers to system property derivative.

Thermodynamics Second Law

There are several definitions of the second law. No matter which definition is used to describe the second law it will end in a mathematical form. The most common mathematical form is Clausius inequality which state that

$$\oint \frac{\delta Q}{T} \geq 0 \tag{2.21}$$

The integration symbol with the circle represent integral of cycle (therefore circle) of system which returns to the same condition. If there is no lost, it is referred as a reversible process and the inequality change to equality.

$$\oint \frac{\delta Q}{T} = 0 \tag{2.22}$$

The last integral can go though several states. These states are independent of the path the system goes through. Hence, the integral is independent of the path. This observation leads to the definition of entropy and designated as S and the derivative of entropy is

$$ds \equiv \left(\frac{\delta Q}{T}\right)_{\text{rev}} \tag{2.23}$$

Performing integration between two states results in

$$S_2 - S_1 = \int_1^2 \left(\frac{\delta Q}{T}\right)_{\text{rev}} = \int_1^2 dS \tag{2.24}$$

One of the conclusions that can be drawn from this analysis is for reversible and adiabatic process $dS = 0$. Thus, the process in which it is reversible and adiabatic, the entropy remains constant and referred to as isentropic process. It can be noted that there is a possibility that a process can be irreversible and the right amount of heat transfer to have zero change entropy change. Thus, the reverse conclusion that zero change of entropy leads to reversible process, isn't correct.

For reversible process equation (2.22) can be written as

$$\delta Q = T dS \tag{2.25}$$

and the work that the system is doing on the surroundings is

$$\delta W = P dV \tag{2.26}$$

Substituting equations (2.25) (2.26) into (2.20) results in

$$T dS = d\, E_U + P dV \tag{2.27}$$

Even though the derivation of the above equations were done assuming that there is no change of kinetic or potential energy, it still remains valid for all situations. Furthermore, it can be shown that it is valid for reversible and irreversible processes.

Enthalpy

It is a common practice to define a new property, which is the combination of already defined properties, the enthalpy of the system.

$$H = E_U + PV \tag{2.28}$$

The specific enthalpy is enthalpy per unit mass and denoted as, h.

Or in a differential form as

$$dH = dE_U + dP\,V + P\,dV \tag{2.29}$$

Combining equations (2.28) the (2.27) yields

$$\boxed{TdS = dH - VdP} \tag{2.30}$$

For isentropic process, equation (2.27) is reduced to $dH = VdP$. The equation (2.27) in mass unit is

$$Tds = du + Pdv = dh - \frac{dP}{\rho} \tag{2.31}$$

when the density enters through the relationship of $\rho = 1/v$.

Specific Heats

The change of internal energy and enthalpy requires new definitions. The first change of the internal energy and it is defined as the following

$$\boxed{C_v \equiv \left(\frac{\partial E_u}{\partial T}\right)} \tag{2.32}$$

And since the change of the enthalpy involve some kind of work, it is defined as

$$\boxed{C_p \equiv \left(\frac{\partial h}{\partial T}\right)} \tag{2.33}$$

The ratio between the specific pressure heat and the specific volume heat is called the ratio of the specific heats and it is denoted as, k.

$$\boxed{k \equiv \frac{C_p}{C_v}} \tag{2.34}$$

For liquid metal used in die casting, the ratio of the specific heats is bite higher than one (1) and therefore the difference between them is almost zero and therefore referred as C.

Equation of state

Equation of state is a relation between state variables. Normally the relationship of temperature, pressure, and specific volume define the equation of state for gases. The simplest equation of state referred to as ideal gas and it is defined as

$$P = \rho R T \tag{2.35}$$

Application of Avogadro's law, that "all gases at the same pressures and temperatures have the same number of molecules per unit of volume," allows the calculation of a "universal gas constant." This constant to match the standard units results in

$$\bar{R} = 8.3145 \frac{kj}{kmol\ K} \tag{2.36}$$

Thus, the specific gas can be calculated as

$$R = \frac{\bar{R}}{M} \tag{2.37}$$

The specific constants for select gas at 300K is provided in table 2.1.

From equation (2.35) of state for perfect gas it follows

$$d(Pv) = R dT \tag{2.38}$$

For perfect gas

$$dh = dE_u + d(Pv) = dE_u + d(RT) = f(T) \text{ (only)} \tag{2.39}$$

From the definition of enthalpy it follows that

$$d(Pv) = dh - dE_u \tag{2.40}$$

Utilizing equation (2.38) and substituting into equation (2.40) and dividing by dT yields

$$C_p - C_v = R \tag{2.41}$$

This relationship is valid only for ideal/perfect gases.

The ratio of the specific heats can be expressed in several forms as

$$\boxed{C_v = \frac{R}{k-1}} \tag{2.42}$$

Table -2.1. Properties of Various Ideal Gases [300K]

Gas	Chemical Formula	Molecular Weight	R $\left[\frac{kj}{KgK}\right]$	C_v $\left[\frac{kj}{KgK}\right]$	C_P $\left[\frac{kj}{KgK}\right]$	k
Air	-	28.970	0.28700	1.0035	0.7165	1.400
Argon	Ar	39.948	0.20813	0.5203	0.3122	1.400
Butane	C_4H_{10}	58.124	0.14304	1.7164	1.5734	1.091
Carbon Dioxide	CO_2	44.01	0.18892	0.8418	0.6529	1.289
Carbon Monoxide	CO	28.01	0.29683	1.0413	0.7445	1.400
Ethane	C_2H_6	30.07	0.27650	1.7662	1.4897	1.186
Ethylene	C_2H_4	28.054	0.29637	1.5482	1.2518	1.237
Helium	He	4.003	2.07703	5.1926	3.1156	1.667
Hydrogen	H_2	2.016	4.12418	14.2091	10.0849	1.409
Methane	CH_4	16.04	0.51835	2.2537	1.7354	1.299
Neon	Ne	20.183	0.41195	1.0299	0.6179	1.667
Nitrogen	N_2	28.013	0.29680	1.0416	0.7448	1.400
Octane	C_8H_{18}	114.230	0.07279	1.7113	1.6385	1.044
Oxygen	O_2	31.999	0.25983	0.9216	0.6618	1.393
Propane	C_3H_8	44.097	0.18855	1.6794	1.4909	1.327
Steam	H_2O	18.015	0.48152	1.8723	1.4108	1.327

$$C_p = \frac{k\,R}{k-1} \tag{2.43}$$

The specific heats ratio, k value ranges from unity to about 1.667. These values depend on the molecular degrees of freedom (more explanation can be obtained in Van Wylen "F. of Classical thermodynamics.") The values of several gases can be approximated as ideal gas and are provided in Table (2.1).

The entropy for ideal gas can be simplified as the following

$$s_2 - s_1 = \int_1^2 \left(\frac{dh}{T} - \frac{dP}{\rho T}\right) \tag{2.44}$$

Using the identities developed so far one can find that

$$s_2 - s_1 = \int_1^2 C_p \frac{dT}{T} - \int_1^2 \frac{R\,dP}{P} = C_p \ln \frac{T_2}{T_1} - R \ln \frac{P_2}{P_1} \tag{2.45}$$

Or using specific heats ratio equation (2.45) transformed into

$$\frac{s_2 - s_1}{R} = \frac{k}{k-1} \ln \frac{T_2}{T_1} - \ln \frac{P_2}{P_1} \tag{2.46}$$

For isentropic process, $\Delta s = 0$, the following is obtained

$$\ln \frac{T_2}{T_1} = \ln \left(\frac{P_2}{P_1} \right)^{\frac{k-1}{k}} \tag{2.47}$$

There are several famous identities that results from equation (2.47) as

$$\boxed{\frac{T_2}{T_1} = \left(\frac{P_2}{P_1} \right)^{\frac{k-1}{k}} = \left(\frac{P_2}{P_1} \right)^{k-1}} \tag{2.48}$$

The ideal gas model is a simplified version of the real behavior of real gas. The real gas has a correction factor to account for the deviations from the ideal gas model. This correction factor is referred to as the compressibility factor and defined as

$$\boxed{Z = \frac{P\,V}{R\,T}} \tag{2.49}$$

Control Volume

The control volume was introduced by L. Euler[1] In the control volume (c.v) the focus is on specific volume which mass can enter and leave. The simplest c.v. is when the boundaries are fixed and it is referred to as the *Non–deformable c.v.*. The conservation of mass to such system can be reasonably approximated by

$$\frac{d}{dt} \int_{V_{c.v.}} \rho dV = - \int_{S_{c.v.}} \rho V_{rn} dA \tag{2.50}$$

This equation states the change in the volume came from the difference of masses being added through the boundary.

put two examples of simple for mass conservation.

For deformable c.v.

$$\frac{d}{dt} \int_{V_{c.v.}} \rho dV = \int_{V_{c.v.}} \frac{d\rho}{dt} dV + \int_{S_{c.v.}} \rho V_{rn} dA \tag{2.51}$$

[1] A blind man known as the master of calculus, made his living by being a tutor, can you imagine he had eleven kids: where he had the time and energy to develop all the great theory and mathematics.

2.3.3 Momentum Equation

The second Newton law of motion is written mathematically as

$$\Sigma F = \frac{D}{Dt} mV \tag{2.52}$$

This explanation, of course, for fluid particles can be written as

$$\Sigma F = \frac{D}{Dt} \int_{V_{sys}} V \rho dV \tag{2.53}$$

or more explicitly it can be written as

$$\Sigma F = \frac{d}{dt} \int_{V_{c.v.}} \rho V dV + \int_{A_{c.v.}} \rho V \cdot V_{rn} dA \tag{2.54}$$

2.3.4 Compressible flow

This material is extensive and requires a semester for student to have good understanding of this complex material. Yet to give very minimal information is seems to to be essential to the understanding of the venting design. The summary material here is derived from the book "Fundamentals of Compressible Flow Mechanics."

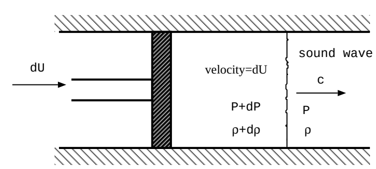


Fig. -2.3. A very slow moving piston in a still gas.

2.3.5 Speed of Sound

The speed of sound is a very important parameter in the die casting process because it effects and explains the choking in the die casting process. What is the speed of the small disturbance $+$as it travels in a "quiet" medium? This velocity is referred to as the speed of sound. To answer this question, consider a piston moving from the left to the right at a relatively small velocity (see Figure 2.3). The information that the piston is moving passes thorough a single "pressure pulse." It is assumed that if the velocity of the piston is infinitesimally small, the pulse will be infinitesimally small. Thus, the pressure and density can be assumed to be continuous.

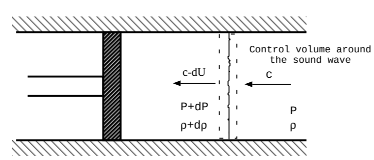


Fig. -2.4. Stationary sound wave and gas moves relative to the pulse.

It is convenient to look at a control volume which is attached to a pressure pulse. Applying the mass balance yields

$$\rho c = (\rho + d\rho)(c - dU) \tag{2.55}$$

or when the higher term $dUd\rho$ is neglected yields

$$\rho dU = cd\rho \Longrightarrow dU = \frac{cd\rho}{\rho} \tag{2.56}$$

From the energy equation (Bernoulli's equation), assuming isentropic flow and neglecting the gravity results

$$\frac{(c - dU)^2 - c^2}{2} + \frac{dP}{\rho} = 0 \tag{2.57}$$

neglecting second term (dU^2) yield

$$-cdU + \frac{dP}{\rho} = 0 \tag{2.58}$$

Substituting the expression for dU from equation (2.56) into equation (2.58) yields

$$c^2 \left(\frac{d\rho}{\rho}\right) = \frac{dP}{\rho} \Longrightarrow c^2 = \frac{dP}{d\rho} \tag{2.59}$$

It is shown in the book "Fundamentals of Compressible Fluid Mechanics" that relationship between n, Z and k is

$$n = \overbrace{\frac{C_p}{C_v}}^{k} \left(\frac{z + T \left(\frac{\partial z}{\partial T}\right)_\rho}{z + T \left(\frac{\partial z}{\partial T}\right)_P} \right) \tag{2.60}$$

Note that n approaches k when $z \rightarrow 1$ and when z is constant. The speed of sound for a real gas can be obtained in similar manner as for an ideal gas

$$\frac{dP}{d\rho} = nzRT \tag{2.61}$$

Speed of Sound in Almost Incompressible Liquid

Even liquid metal *normally* is assumed to be incompressible but in reality it has a small and important compressible aspect. The ratio of the change in the fractional volume to pressure or compression is referred to as the bulk modulus of the material. The mathematical definition of bulk modulus is as follows

$$B = \rho \frac{dP}{d\rho} \tag{2.62}$$

In physical terms it can be written as

$$c = \sqrt{\frac{elastic\ property}{inertial\ property}} = \sqrt{\frac{B}{\rho}} \tag{2.63}$$

In summary, the speed of sound in liquid metals is about 5 times faster than the speed of sound in gases in the chamber.

2.3.6 Choked Flow

In this section a discussion on a steady state flow through a smooth and continuous area flow rate is presented which include the flow through a converging–diverging nozzle. The isentropic flow models are important because of two main reasons:

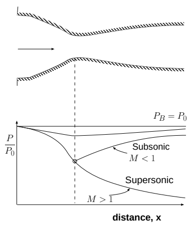


Fig. -2.5. Flow of a compressible substance (gas) through a converging–diverging nozzle.

Stagnation State for Ideal Gas Model

It is assumed that the flow is one–dimensional. Figure (2.5) describes a gas flow through a converging–diverging nozzle. It has been found that a theoretical state known as the stagnation state is very useful in which the flow is brought into a complete motionless condition in isentropic process without other forces (e.g. gravity force). Several properties can be represented by this theoretical process which include temperature, pressure, and density etc and denoted by the subscript "0."

A dimensionless velocity and it is referred as Mach number for the ratio of velocity to speed of sound as

$$M \equiv \frac{U}{c} \tag{2.64}$$

The temperature ratio reads

$$\frac{T_0}{T} = 1 + \frac{k-1}{2} M^2 \tag{2.65}$$

The ratio of stagnation pressure to the static pressure can be expressed as the function of the temperature ratio because of the isentropic relationship as

$$\frac{P_0}{P} = \left(\frac{T_0}{T}\right)^{\frac{k}{k-1}} = \left(1 + \frac{k-1}{2} M^2\right)^{\frac{k}{k-1}} \tag{2.66}$$

In the same manner the relationship for the density ratio is

$$\frac{\rho_0}{\rho} = \left(\frac{T_0}{T}\right)^{\frac{1}{k-1}} = \left(1 + \frac{k-1}{2} M^2\right)^{\frac{1}{k-1}} \tag{2.67}$$

A new useful definition is introduced for the case when $M = 1$ and denoted by superscript "$*$." The special case of ratio of the star values to stagnation values are dependent only on the heat ratio as the following:

$$\frac{T^*}{T_0} = \frac{{c^*}^2}{{c_0}^2} = \frac{2}{k+1} \tag{2.68}$$

and

$$\frac{P^*}{P_0} = \left(\frac{2}{k+1}\right)^{\frac{k}{k-1}} \tag{2.69}$$

$$\frac{\rho^*}{\rho_0} = \left(\frac{2}{k+1}\right)^{\frac{1}{k-1}} \tag{2.70}$$

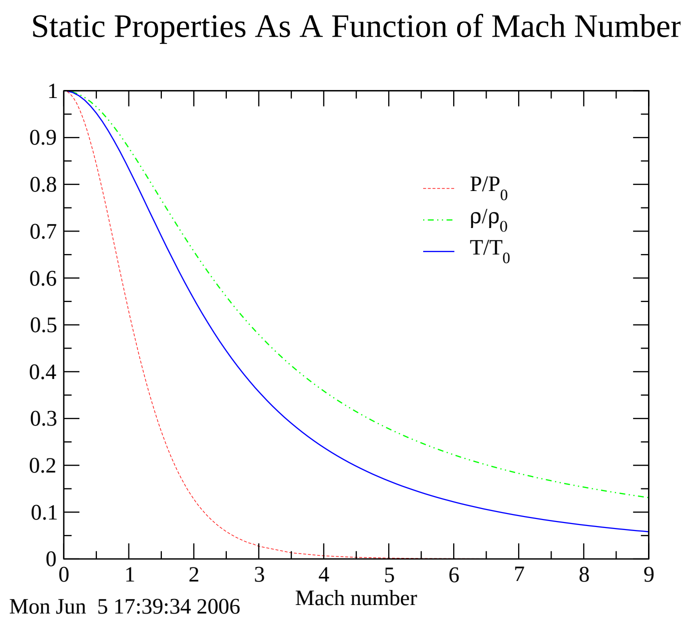


Fig. -2.6. The stagnation properties as a function of the Mach number, k=1.4

The definition of the star Mach is ratio of the velocity and star speed of sound at $M = 1$.

The flow in a converging–diverging nozzle has two models: First is isentropic and adiabatic model. Second is isentropic and isothermal model. Clearly, the stagnation temperature, T_0, is constant through the adiabatic flow because there isn't heat transfer. Therefore, the stagnation pressure is also constant through the flow because of the isentropic flow. Conversely, in mathematical terms, equation (2.65) and equation (2.66) are the same. If the right hand side is constant for one variable, it is constant for the other. In the same argument, the stagnation density is constant through the flow. Thus, knowing the Mach number or the temperature will provide all that is needed to find the other properties. The only properties that need to be connected are the cross section area and the Mach number. Examination of the relation between properties can then be carried out.

The Properties in the Adiabatic Nozzle

When there is no external work and heat transfer, the energy equation, reads

$$dh + U dU = 0 \tag{2.71}$$

Differentiation of continuity equation, $\rho A U = \dot{m} = constant$, and dividing by the continuity equation reads

$$\frac{d\rho}{\rho} + \frac{dA}{A} + \frac{dU}{U} = 0 \tag{2.72}$$

The thermodynamic relationship between the properties can be expressed as

$$T ds = dh - \frac{dP}{\rho} \tag{2.73}$$

For isentropic process $ds \equiv 0$ and combining equations (2.71) with (2.73) yields

$$\frac{dP}{\rho} + U dU = 0 \tag{2.74}$$

Differentiation of the equation state (perfect gas), $P = \rho R T$, and dividing the results by the equation of state ($\rho R T$) yields

$$\frac{dP}{P} = \frac{d\rho}{\rho} + \frac{dT}{T} \tag{2.75}$$

Obtaining an expression for dU/U from the mass balance equation (2.72) and using it in equation (2.74) reads

$$\frac{dP}{\rho} - U^2 \overbrace{\left[\frac{dA}{A} + \frac{d\rho}{\rho}\right]}^{\frac{dU}{U}} = 0 \tag{2.76}$$

Rearranging equation (2.76) so that the density, ρ, can be replaced by the static pressure, dP/ρ yields

$$\frac{dP}{\rho} = U^2 \left(\frac{dA}{A} + \frac{d\rho}{\rho}\frac{dP}{dP}\right) = U^2 \left(\frac{dA}{A} + \overbrace{\frac{d\rho}{dP}}^{\frac{1}{c^2}} \frac{dP}{\rho}\right) \tag{2.77}$$

Recalling that $dP/d\rho = c^2$ and substitute the speed of sound into equation (2.77) to obtain

$$\frac{dP}{\rho}\left[1-\left(\frac{U}{c}\right)^2\right] = U^2\frac{dA}{A} \tag{2.78}$$

Or in a dimensionless form

$$\frac{dP}{\rho}\left(1-M^2\right) = U^2\frac{dA}{A} \tag{2.79}$$

Equation (2.79) is a differential equation for the pressure as a function of the cross section area. It is convenient to rearrange equation (2.79) to obtain a variables separation form of

$$dP = \frac{\rho U^2}{A}\,\frac{dA}{1-M^2} \tag{2.80}$$

Before going further in the mathematical derivation it is worth while to look at the physical meaning of equation (2.80). The term $\rho U^2/A$ is always positive (because all the three terms can be only positive). Now, it can be observed that dP can be positive or negative depending on the dA and Mach number. The meaning of the sign change for the pressure differential is that the pressure can increase or decrease. It can be observed that the critical Mach number is one. If the Mach number is larger than one than dP has opposite sign of dA. If Mach number is smaller than one dP and dA have the same sign. For the subsonic branch $M < 1$ the term $1/(1-M^2)$ is positive hence

$$dA > 0 \Longrightarrow dP > 0$$
$$dA < 0 \Longrightarrow dP < 0$$

From these observations the trends are similar to those in incompressible fluid. An increase in area results in an increase of the static pressure (converting the dynamic pressure to a static pressure). Conversely, if the area decreases (as a function of x) the pressure decreases. Note that the pressure decrease is larger in compressible flow compared to incompressible flow.

For the supersonic branch $M > 1$, the phenomenon is different. For $M > 1$ the term $1/1-M^2$ is negative and change the character of the equation.

$$dA > 0 \Rightarrow dP < 0$$
$$dA < 0 \Rightarrow dP > 0$$

This behavior is opposite to incompressible flow behavior.

For the special case of $M = 1$ (sonic flow) the value of the term $1-M^2 = 0$ thus mathematically $dP \to \infty$ or $dA = 0$. Since physically dP can increase only in a finite amount it must be that $dA = 0$.It must also be noted that when $M = 1$ occurs only when $dA = 0$. However, the opposite, not necessarily means that when $dA = 0$ that $M = 1$. In that case, it is possible that $dM = 0$ thus the diverging side is in the subsonic branch and the flow isn't choked.

Isentropic Isothermal Flow Nozzle

In this section, the other extreme case model where the heat transfer to the gas is perfect, (e.g. Eckert number combination is very small) is presented. Again in reality the heat transfer is somewhere in between the two extremes. So, knowing the two limits provides a tool to examine where the reality should be expected. The perfect gas model is again assumed. In isothermal process the perfect gas model reads

$$P = \rho R T \rightsquigarrow dP = d\rho R T \tag{2.81}$$

Substituting equation (2.81) into the momentum equation[2] yields

$$U dU + \frac{R T dP}{P} = 0 \tag{2.82}$$

Integration of equation (2.82) yields the Bernoulli's equation for ideal gas in isothermal process which reads

$$\rightsquigarrow \frac{{U_2}^2 - {U_1}^2}{2} + R T \ln \frac{P_2}{P_1} = 0 \tag{2.83}$$

Then the stagnation velocity is

$$U = \sqrt{2 R T \ln \frac{P}{P_0}} \tag{2.84}$$

It can be shown that the pressure ratio is

$$\frac{P_2}{P_1} = e^{\frac{k({M_1}^2 - {M_2}^2)}{2}} = \left(\frac{e^{{M_1}^2}}{e^{{M_2}^2}} \right)^{\frac{k}{2}} \tag{2.85}$$

As opposed to the adiabatic case ($T_0 = constant$) in the isothermal flow the stagnation temperature ratio can be expressed

$$\frac{T_{01}}{T_{02}} = \overbrace{\frac{\cancel{T_1}}{\cancel{T_2}}}^{1} \frac{\left(1 + \frac{k-1}{2}{M_1}^2\right)}{\left(1 + \frac{k-1}{2}{M_2}^2\right)} = \frac{\left(1 + \frac{k-1}{2}{M_1}^2\right)}{\left(1 + \frac{k-1}{2}{M_2}^2\right)} \tag{2.86}$$

Combining equation mass conservation with equation (2.85) yields

$$\frac{A_2}{A_1} = \frac{M_1}{M_2} \left(\frac{\mathrm{e}^{{M_2}^2}}{\mathrm{e}^{{M_1}^2}} \right)^{\frac{k}{2}} \tag{2.87}$$

[2]The one dimensional momentum equation for steady state is $U dU/dx = -dP/dx + 0$(other effects) which are neglected here.

The change in the stagnation pressure can be expressed as

$$\frac{P_{02}}{P_{01}} = \frac{P_2}{P_1}\left(\frac{1+\frac{k-1}{2}{M_2}^2}{1+\frac{k-1}{2}{M_1}^2}\right)^{\frac{k}{k-1}} = \left[\frac{\mathrm{e}^{{M_1}^2}}{\mathrm{e}^{{M_1}^2}}\right]^{\frac{k}{2}} \tag{2.88}$$

The critical point, at this stage, is unknown (at what Mach number the nozzle is choked is unknown) so there are two possibilities: the choking point or $M = 1$ to normalize the equation. Here the critical point defined as the point where$M = 1$ so results can be compared to the adiabatic case and denoted by star. Again it has to be emphasized that this critical point is not really related to physical critical point but it is only an arbitrary definition. The true critical point is when flow is choked and the relationship between two will be presented.

The critical pressure ratio can be obtained from (2.85) to read

$$\frac{P}{P^*} = \frac{\rho}{\rho^*} = \mathrm{e}^{\frac{(1-M^2)k}{2}} \tag{2.89}$$

Equation (2.87) is reduced to obtained the critical area ratio writes

$$\frac{A}{A^*} = \frac{1}{M}\mathrm{e}^{\frac{(1-M^2)k}{2}} \tag{2.90}$$

Similarly the stagnation temperature reads

$$\frac{T_0}{{T_0}^*} = \frac{2\left(1+\frac{k-1}{2}{M_1}^2\right)}{k+1}^{\frac{k}{k-1}} \tag{2.91}$$

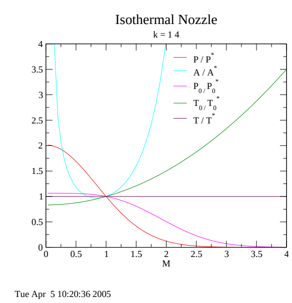


Fig. -2.7. Various ratios as a function of Mach number for isothermal Nozzle

Finally, the critical stagnation pressure reads

$$\frac{P_0}{{P_0}^*} = \mathrm{e}^{\frac{(1-M^2)k}{2}}\left(\frac{2\left(1+\frac{k-1}{2}{M_1}^2\right)}{k+1}\right)^{\frac{k}{k-1}} \tag{2.92}$$

The maximum value of stagnation pressure ratio is obtained when $M = 0$ at which is

$$\left.\frac{P_0}{{P_0}^*}\right|_{M=0} = \mathrm{e}^{\frac{k}{2}}\left(\frac{2}{k+1}\right)^{\frac{k}{k-1}} \tag{2.93}$$

For specific heats ratio of $k = 1.4$, this maximum value is about two. It can be noted that the stagnation pressure is monotonically reduced during this process.

Of course in isothermal process $T = T^*$. All these equations are plotted in Figure (2.7). From the Figure 2.7 it can be observed that minimum of the curve A/A^* isn't on $M = 1$. The minimum of the curve is when area is minimum and at the point where the flow is choked. It should be noted that the stagnation temperature is not constant as in the adiabatic case and the critical point is the only one constant.

The mathematical procedure to find the minimum is simply taking the derivative and equating to zero as the following

$$\frac{d\left(\frac{A}{A^*}\right)}{dM} = \frac{kM^2 e^{\frac{k(M^2-1)}{2}} - e^{\frac{k(M^2-1)}{2}}}{M^2} = 0 \tag{2.94}$$

Equation (2.94) simplified to

$$kM^2 - 1 = 0 \leadsto M = \frac{1}{\sqrt{k}} \tag{2.95}$$

"The shear, S, at the ingate is determined by the average velocity, U, of the liquid and by the ingate thickness, t. Dimensional analysis shows that is directly proportional to (U/ℓ). The constant of proportionality is difficult to determine, . . .[1]"

Murray, CSIRO Australia

CHAPTER 3

Dimensional Analysis

One of the important tools to understand the die casting process is dimensional analysis. Fifty years ago, this method transformed the fluid mechanics/heat transfer into a "uniform" understanding. This book attempts to introduce to the die casting industry this established method[2]. Experimental studies will be "expanded/generalized" as it was done in convective heat transfer. It is hoped that as a result, separate sections for aluminum, zinc, and magnesium will not exist anymore in die casting conferences. This chapter is based partially on Dr. Eckert's book, notes, and the article on dimensional analysis applied to die casting. Several conclusions are derived from this analysis and they will be presented throughout this chapter. This material can bring great benefit to researchers who want to built their research on a solid foundation. For those who are dealing with the numerical research/calculation, it is useful to learn when some parameters should be taken into account and why.

[1] Citing "The Design of feed systems for thin walled zinc high pressure die castings," Metallurgical and materials transactions B Vol. 27B, February 1996, pp. 115–118. This excerpt is an excellent example of poor research and poor understanding. This "unknown" constant is called viscosity (see Basics of Fluid Mechanics in Potto series. Here, a discussion on some specific mistakes were presented in that paper (which are numerous). Dimensional analysis is a tool which can take "cluttered" and meaningless paper such as the above and turn them into something with real value. As proof of their model, the researchers have mentioned two unknown companies that their model is working. What a nice proof! Are the physics laws really different in Australia?

[2] Actually, Prof. E.R.G. Eckert introduced the dimensional analysis to the die casting long before. The author is his zealous disciple, all the credit should go to Eckert. Of course, all the mistakes are the author's and none of Dr. Eckert's. All the typos in Eckert's paper were this author's responsibility for which he apologizes.

3.0.7 How The Dimensional Analysis Work

In dimensional analysis, the number of the effecting parameters is reduced to a minimum by replacing the dimensional parameters by dimensionless parameters. Some researchers point out that the chief advantage of this analysis is "to obtain experimental results with a minimum amount of labor, results in a form having maximum utility" [18, pp. 395]. The dimensional analysis has several other advantages which include; 1)increase of understanding, 2) knowing what is important, and 3) compacting the presentation[3]. The advantage of compact of presentation allows one to "see" the big picture with minimal effort.

Dimensionless parameters are parameters which represent a ratio which does not have a physical dimension. The experimental study assists to solve problems when the solution of the governing equation cannot be obtained. To achieve this, experiments are designed to be "similar" to the situations which need to be solved or simulated. The base for this concept is mathematical. Two different sets of phenomena will produce a similar result if the governing differential equations with boundaries conditions are similar. The actual experiments are difficult to carry out in many cases. Thus, design experiments with the same governing differential equations as the actual phenomenon is the solution. This similarity does not necessarily mean that the experiments have to be carried exactly as studied phenomena. It is enough that the main dimensionless parameters are similar, since the minor dimensional parameters, in many cases, are insignificant. For example, a change in Reynolds number is insignificant since a change in Reynolds number in a large range does not affect the friction factor.

An example of the similarity applied to the die cavity is given in the section 3.5. Researchers in casting in general and die casting in particular do not utilize this method. For example, after the Russians [6] introduced the water analogy method (in casting) in the 40s all the experiments such as Wallace, CSIRO, etc. conducted poorly designed experiments. For example, Wallace record the Reynolds and Froude number without attempting to match the governing equations. Another example is the experimental study of Gravity Tiled Die Casting (low pressure die casting) performed by Nguyen's group in 1986 comparing two parameters Re and We. Flow of "free" falling, the velocity is a function of the height ($U \sim \sqrt{gH}$). Hence, the equation $Re_{model} = Re_{actual}$ should lead only to $H_{model} \equiv H_{actual}$ and not to any function of U_{model}/U_{actual}. The value of U_{model}/U_{actual} is actually constant for the same height ratio. The Wallace experiments with Reynolds number matching does not lead to matching of similar governing equations. Many other important parameters which control the governing equations are not simulated [26]. The governing equations in these cases include several other important parameters which have not been controlled

[3]The importance of compact presentation is attributed to Prof. M. Bentwitch who was mentor to many including the author during his masters studies.

or even measured, monitored, and simulated[4]. Moreover, the Re number is controlled by the flow rate and the characteristics of the ladle opening and not as in the pressurized pipe flow as the authors assumed.

3.1 Introduction

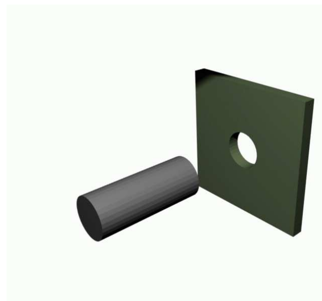


Fig. -3.1. Rod into the hole example

Lets take a trivial example of fitting a rode into a circular hole (see Figure 3.1). To solve this problem, it is required to know two parameters; 1) the diameter of the rode and 2) the diameter of the hole. Actually, it is required to have only one parameter, the ratio of the rode diameter to the hole diameter. The ratio is a dimensionless number and with this number one can say that for a ratio larger than one, the rode will not enter the hole; and ratio smaller than one, the rod is too small. Only when the ratio is equal to one, the rode is said to be fit. This allows one to draw the situation by using only one coordinate. Furthermore, if one wants to deal with tolerances, the dimensional analysis can easily be extended to say that when the ratio is equal from 0.99 to 1.0 the rode is fitting, and etc. If one were to use the two diameters description, he will need more than this simple sentence to describe it.

In the preceding simplistic example, the advantages are minimal. In many real problems, including the die casting process, this approach can remove clattered views and put the problem into focus. It also helps to use information from different problems to a “similar” situation. Throughout this book the reader will notice that the systems/equations are converted to a dimensionless form to augment understanding.

3.2 The Die Casting Process Stages

The die casting process can be broken into many separated processes which are controlled by different parameters. The simplest division of the process for a cold chamber is the following: 1) filling the shot sleeve, 2) slow plunger velocity, 3) filling the runner system 4) filling the cavity and overflows, and 5) solidification process (also referred as intensification process). This division into such sub–processes results in a clear picture

[4]Besides many conceptual physical mistakes, the authors have a conceptual mathematical mistake. They tried to achieve the same Re and Fr numbers in the experiments as in reality for low pressure die casting. They derived an equation for the velocity ratio based on equal Re numbers (model and actual). They have done the same for Fr numbers. Then they equate the velocity ratio based on equal Re to velocity ratio based on equal Fr numbers. However, velocity ratio based on equal Re is a constant and does vary with the tunnel dimension (as opposed to distance from the starting point). The fact that these ratios have the same symbols do not mean that they are really the same. These two ratios are different and cannot be equated.

on each process. On one hand, in processes 1 to 3, it is desirable to have a minimum heat transfer/solidification to take place for obvious reasons. On the other hand, in the rest of the processes, the solidification is the major concern.

In die casting, the information and conditions do not travel upstream. For example, the turbulence does not travel from some point at the cavity to the runner and of–course, to the shot sleeve. This kind of relationship is customarily denoted as a parabolic process (because in mathematics the differential equations describe these kind of cases as parabolic). To a larger extent it is true in die casting. The pressure in the cavity does not affect the flow in the sleeve or the runner if the vent system is well designed. In other words, the design of the pQ^2 diagram is not controlled by down–stream conditions. Another example, the critical slow plunger velocity is not affected by the air/gas flow/pressure in the cavity. In general, the turbulence generated down–stream does not travel up–stream in this process. One has to restrict this characterization to some points. One point is particularly mentioned here: The poor design of the vent system affects the pressure in the cavity and therefore the effects do travel down stream. For example, the pQ^2 diagram calculations are affected by poor vent system design.

3.2.1 Filling the Shot Sleeve

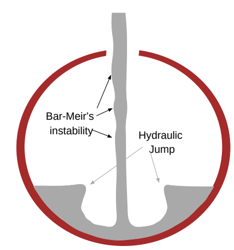


Fig. -3.2. Hydraulic jump in the shot sleeve.

The flow from the ladle to the shot sleeve did not receive much attention in the die casting research[5] because it is believed that it does not play a significant role. For low pressure die casting, the flow of liquid metal from the ladle through "channel(s)" to the die cavity plays an important role[6]. The importance of the understanding of this process can show us how to minimize the heat transfer, layer created on the sleeve (solidification layer), and sleeve protection from; a) erosion b) plunger problem. The jet itself has no smooth surface and two kinds of instability occurs. The first instability is of Bernoulli's effect and second effect is Bar-Meir's effect that boundary conditions cannot be satisfied for two phase flow. Yet, for die casting process, these two effects (see Figure 3.2 do not change the global flow in the sleeve. At first, the hydraulic jump is created when the liquid metal enters the sleeve. The typical time scale for hydraulic jump creation is almost instant and extremely short as can be shown by the characteristic methods. As the liquid metal level in the sleeve rises, the location of the jump moves closer to the impinging center. At a certain point, the liquid depth level is over the critical depth level and the hydraulic jump disappears. The critical depends on the liquid properties and the ratio of impinging momentum or velocity to the hydraulic static pressure. The impinging momentum impact is proportional to $\rho U^2 \pi r^2$ and hydraulic "pressure" is proportional

[5] Very few papers (~ 0) can be found dealing with this aspect.

[6] Some elementary estimates of fluid mechanics and heat transfer were made by the author and hopefully will be added to this book.

to $\rho\, g\, h\, 2\, \pi\, r\, h$. Where r is the radius of the impinging jet and h is the height of the liquid metal in the sleeve. The above statement leads to

$$U_{critical} \propto \sqrt{\frac{g\, h^2}{r}} \tag{3.1}$$

The critical velocity on the other hand has to be

$$U_critical = g\, h_L \tag{3.2}$$

where h_L is the distance of the ladle to the height of the liquid metal in the sleeve. The height where the hydraulic shock will not exist is

$$h_{critical} \sim \sqrt{r\, h_L} \tag{3.3}$$

This analysis suggests that decreasing the ladle height and/or reducing less mass flow rate (the radius of the jet) result in small critical height. The air entrainment during that time will be discussed in the book "Basic of Fluid Mechanics" in the Multi–Phase flow chapter. At this stage, air bubbles are entrained in the liquid metal which augment the heat transfer. At present, there is an extremely limited knowledge about the heat transfer during this part of the process, and of course less about how to minimize it. However, this analysis suggests that minimizing the ladle height is one of the ways to reduce it.

The heat transfer from liquid metal to the surroundings is affected by the velocity and the flow patterns since the mechanism of heat transfer is changed from a dominated natural convection to a dominated force convection. In addition, the liquid metal jet surface is also affected by heat transfer to some degree by change in the properties.

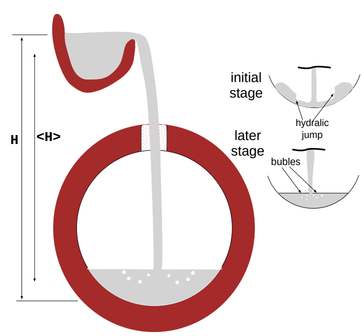


Fig. -3.3. Filling of the shot sleeve.

Heat Transferred to the Jet

The estimate on heat transfer requires some information on jet dynamics. There are two effects that must be addressed; one the average radius and the fluctuation of the radius. As first approximation, the average jet radius changes due to the velocity change. For laminar flow, (for simplicity assume plug flow) the velocity function is $\sim \sqrt{x}$ where x is the distance from the ladle. For constant flow rate, neglecting the change of density, the radius will change as $r \sim 1/\sqrt[4]{x}$. Note that this relationship is not valid when it is very near the ladle proximity $(r/x \sim 0)$. The heat transfer increases as a function of x for these two reasons.

The second effect is jet radius fluctuations. Consider this, the jet leaves the ladle in a plug flow. Due to air friction, the shear stress changes the velocity profile to parabolic. For simple assumption of steady state(it is not steady state), the momentum equation which governs the liquid metal is

$$\rho\left(\overbrace{\frac{\partial u_z}{\partial t}}^{\text{assume } 0} + \overbrace{u_r\frac{\partial u_z}{\partial r}}^{\text{constant}}\right) = \mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u_z}{\partial r}\right)\right] \tag{3.4}$$

Equation (3.4) is in simplified equation form for the gas and liquid phases. Thus, there are two equations that needs to be satisfied simultaneously; one for the gas side and one for the liquid side. Even neglecting several terms for this discussion, it clear that both equations are second order differential equations which have different boundary conditions. Any second order differential equation requires two different boundary conditions. Requirement to satisfy additional boundary condition can be achieved. Thus from physical point of view, second order differential equation which needs to satisfy three boundary conditions is not possible, Thus there must be some wrong either with the governing equation or with the boundary conditions. In this case, the two governing equations must satisfy five (5) different boundary conditions. These boundary conditions are as follows: 1) summitry at $r = 0$, 2) identical liquid metal and air velocities at the interface, 3) identical shear stress at the interface, 4) zero velocity at infinity for the air, and 5) zero shear stress for the air at the infinity. These requirements cannot be satisfied if the interface between the liquid metal and the gas is a straight line.

The heat transfer to the sleeve in the impinging area is significant but at present only very limited knowledge is available due to complexity.

3.2.2 Plunger Slow Moving Part

Fluid Mechanics

The main point is the estimate for energy dissipation. The dissipation is proportional to $\mu < U >^2 L$. Where the strange velocity, $< U >$ is averaged kinetic velocity provided by jet. This kinetic energy is at most the same as potential energy of liquid metal in the ladle. The potential energy in the ladle is $< H >\ m\,g$ where $< H >$ is averaged height see Figure 3.3. The averaged velocity in the shot sleeve is $\sqrt{2\,g\,< H >}$

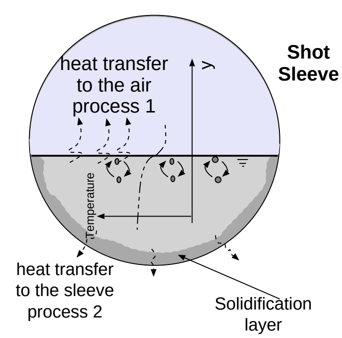


Fig. -3.4. Heat transfer processes in the shot sleeve.

The rate energy dissipation can be estimated as $\mu\left(\frac{<U>}{R}\right)^2 \pi\,R\,L$ as L is the length of shot sleeve. The shear stress is assumed to occur equally in the volume of liquid metal in the sleeve. This assumption of shear stress grossly under estimates the dissipation.

The actual dissipation is larger due to the larger velocity gradients. The estimated time is then

Heat Transfer

In this section, the solidification effects are examined. One of the assumptions in the analysis of the critical slow plunger velocity is that the solidification process does not play an important role (see Figure 3.4). The typical time for heat to penetrate a typical layer in air/gas phase is in the order of minutes. Moreover, the density of the air/gas is 3 order magnitude smaller than liquid metal. Hence, most of the resistance to heat transfer is in the gas phase. Additionally, it has been shown that the liquid metal surface is continuously replaced by slabs of material below the surface which is known in scientific literature as the renewal surface theory. Thus, the main heat transfer mechanism is through the liquid metal to the sleeve. The heat transfer rate for a very thin solidified layer can be approximated as

$$Q \sim k_{lm} \frac{\Delta T}{r} \pi \, r \, L \sim L_s \, \pi \, r \, L \, t \, \rho \tag{3.5}$$

Where L_s is the latent heat, k_{lm} is the thermal conductivity of liquid metal and t is the thickness of the solidification layer. Equation (3.6) results in

$$\frac{t}{r} \propto \frac{k_{lm} \, \Delta T}{L_s \, r^2 \, \rho} \tag{3.6}$$

The value for this die casting process in minutes is in the range of 0.01-0.001 after the thickness reaches to 1-2 [mm]. The relative thickness further decreases as the inverse of the square solidified layer increases. If the solidification is less than one percent of the radius, the speed will be very small compared to the speed of the plunger. If the solidification occur as a mushy zone then the heat transfer is reduced further and it is even lower than this estimate and $\frac{\ell}{R} \ll 1$). Therefore, the heat transfer from the liquid metal surface to the air, as shown in Figure 3.4 (mark as process 1), acts as an insulator to the liquid metal.

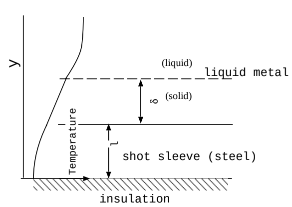


Fig. -3.5. Solidification of the shot sleeve time estimates.

The governing equation in the sleeve is

$$\rho_d {c_p}_d \frac{\partial T}{\partial t} = k_d \left(\frac{\partial^2 T}{\partial y^2} \right) \tag{3.7}$$

where the subscript d denotes the properties of the sleeve material.

Boundary condition between the sleeve and the air/gas is

$$\left. \frac{\partial T}{\partial n} \right|_{y=0} = 0 \tag{3.8}$$

Where n represents the perpendicular direction to the die. Boundary conditions between the liquid metal (solid) and sleeve

$$k_{steel} \left(\frac{\partial T}{\partial y} \right) \bigg|_{y=l} = k_{AL} \left(\frac{\partial T}{\partial y} \right) \bigg|_{y=l} \tag{3.9}$$

The governing equation for the liquid metal (solid phase)

$$\rho_{lm} {c_p}_{lm} \frac{\partial T}{\partial t} = k_{lm} \left(\frac{\partial^2 T}{\partial y^2} \right) \tag{3.10}$$

where lm denotes the properties of the liquid metal. The dissipation and the velocity are neglected due to the change of density and natural convection.

Boundary condition between the phases of the liquid metal is given by

$$v_s \rho_s h_{sf} = k_l \left(\frac{\partial T}{\partial y} \right) \bigg|_{y=l+\delta} - k_s \left(\frac{\partial T}{\partial y} \right) \bigg|_{y=l+\delta} \sim k \frac{\partial (T_l - T_s)}{\partial y} \bigg|_{y=l+\delta} \tag{3.11}$$

h_{sf}	the heat of solidification
ρ_s	liquid metal density at the solid phase
v_n	velocity of the liquid/solid interface
k	conductivity

Neglecting the natural convection and density change, the governing equation in the liquid phase is

$$\rho_l {c_p}_l \frac{\partial T}{\partial t} = k_l \frac{\partial^2 T}{\partial y^2} \tag{3.12}$$

The dissipation function can be assumed to be negligible in this case.

There are three different periods in heat transfer;

1. filling the shot sleeve

2. during the quieting time, and

3. during the plunger movement.

In the first period, heat transfer is relatively very large (major solidification). At present, there is not much known about the fluid mechanics not to say much about the solidification process/heat transfer in fluid mechanics. The second period can be simplified and analyzed as known initial velocity profile. A simplified assumption can be made considering the fact that Pr number is very small (large thermal boundary layer compared to fluid mechanics boundary layer). Additionally, it can be assumed that the natural convection effects are marginal. In the last period, the heat transfer is composed from two zones: 1) behind the jump and 2) ahead of the jump. The heat transfer ahead of the jump is the same as in the second period; while the heat transfer behind the jump is like heat transfer into a plug flow for low Pr number. The heat transfer in such cases have been studied in the past[7].

[7]The reader can refer, for example, to the book "Heat and Mass Transfer" by Eckert and Drake.

3.2.3 Runner system

The flow in the runner system has to be divided into sections; 1) flow with free surface 2) filling the cavity when the flow is pressurized (see Figures 3.6 and 3.7). In the first section the gravity affects the air entrainment. The dominant parameters in this case are Weber number, We and Reynolds number, Re. This phenomenon determines how much metal has to be flushed out. It is well known that the liquid interface cannot be a straight line. Above certain velocity (typical to die casting, high Re number) air leaves streaks of air/gas slabs behind the "front line" as shown in Figure 3.6. These streaks create a low heat transfer zone at the head of the "jet" and "increases" its velocity. The air entrainment created in this case is supposed to be flushed out through the vent system in a proper process design. Unfortunately, at present very little is known about this issue especially the geometry typical to die casting.

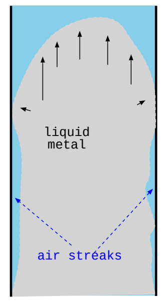


Fig. -3.6. Entrance of liquid metal to the runner.

Gravity Limited in Runner system

In the second phase, the flow in the runner system is pressurized. The typical velocity is large of the range of 10-15 [m/sec]. The typical runner length is in order of 0.1[m]. The velocity due to gravity is $\approx$ 2.5[m/sec]. The Fr number assumes the value $\sim 10^2$ for which gravity play a limited role.

The converging nozzle such as the transition into runner system (which a good die casting engineer should design) tends to reduce the turbulence, if turbulence exists, and can even eliminate it. In that view, the liquid metal enters the runner system as a laminar flow (actually close to a plug flow). For a duct with a typical dimension of 10 [mm] and a mean velocity, U $= 10$[m/sec], (during the second stage), for aluminum die casting, the Reynolds number is:

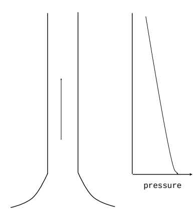


Fig. -3.7. Flow in runner when during pressurizing process.

$$Re = \frac{Ub}{\nu} \approx 5 \times 10^{-7}$$

which is a supercritical flow. However, the flow is probably laminar flow due to the short time.

Another look at turbulence issue: The boundary layer is a function of the time (during the filling period) is of order

$$\delta = 12\nu t$$

The boundary layer in this case can be estimated as[8] the time of the first phase. Anyhow, utilizing the time of 0.01[sec] the viscosity of aluminum in the boundary layer is of the thickness of 0.25[mm] which indicates that flow is laminar.

3.2.4 Die Cavity

All the numerical simulations of die filling are done almost exclusively by assuming that the flow is turbulent and continuous (no two phase flow). In the section 3.3.1 a question about the question whether existence of turbulence is discussed and if so what kind of model is appropriate. Thus, the validity of these numerical models is examined. The liquid metal enters the cavity as a non–continuous flow. According to some researchers, it is preferred that the flow will be atomized (spray). While there is a considerable literature about many geometries none available to typical die casting configurations[9]. The flow can be atomized as either in laminar or turbulent region. The experiments by the author and by others, showed that the flow turns into spray in many cases (See Figures 3.8).

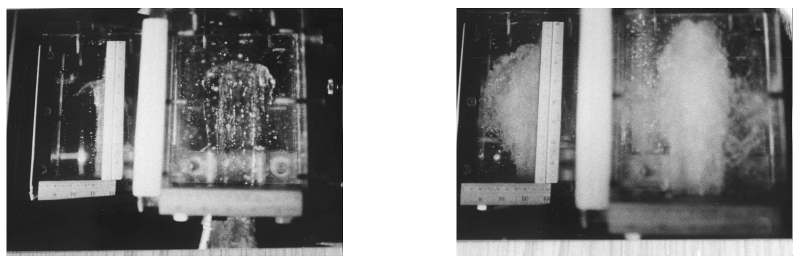


Fig a. *Flow as a jet.* Fig b. *Flow as a spray.*

Fig. -3.8. Typical flow pattern in die casting, jet entering into empty cavity.

In the section 3.4.1 it was shown that the time for atomization is very fast compared with any other process (filling time scale and, of course, the conduction heat transfer or solidification time scales). Atomization requires two streams with a significant velocity difference; stronger surface tension forces against the maintaining stability forces. Numerous experimental studies have shown that better castings are obtained when the injected velocity is above a certain value. This fact alone is enough to convince researchers that the preferred flow pattern is a spray flow. Yet, only a very small number of numerical models exist assuming spray flow and are used for die casting (for example, the paper by Hu at el [22].). Experimental work commonly cited as a "proof"

[8] only during the flow in the runner system, no filling of the cavity

[9] One can just wonder who were the opposition to this research? Perhaps one of the referees as in the Appendix B for the all clues that have been received.

of turbulence was conducted in the mid 60s [30] utilizing water analogy[10]. The "white" spats they observed in their experiments are atomization of the water. Because these experiments were poorly conducted (no similarity to die casting process) the observation/information from these studies is very limited. Yet with this limitation in mind, one can conclude that the spray flow does exist.

Experiments by Fondse et al [16] show that atomization is larger in laminar flow compared to a turbulent flow in a certain range. This fact further creates confusion of what is the critical velocity needed in die casting. Since the experiments which measure the critical velocity were poorly conducted, no reliable information is available on what is the flow pattern and what is the critical velocity[11].

3.2.5 Intensification Period

The two main concerns in this phase is to extract heat from the die and to solidify the liquid metal as aptly as possible to obtain the final shape. Thus, two operational parameters are important; one the (minimum) time for the intensification and two the pressure of the intensification (the clamping force). These two operational parameters can improve casting design to obtain good product.

The main resistance to the heat flow is in the die and the cooling liquid (oil or water based solution). In some parts of the process, the heat is transformed to the cooling liquid via the boiling mechanism. However, the characteristic of boiling heat transfer time to achieve a steady state is larger than the whole process and the typical equations (steady state) for the preferred situation (heat transfer only in the first mode) are not accurate. When there is very limited understanding of so many aspects of the process, the effects of each process on other processes are also cluttered.

3.3 *Special Topics*

3.3.1 Is the Flow in Die Casting Turbulent?

It is commonly assumed that the flow in die casting processes is turbulent in the shot sleeve, runner system, and during the cavity filling. Further, it also assumed that the $k-\epsilon$ model can reasonably represent the turbulence structure. These assumptions are examined herein. The flow can be examined in three zones: 1) the shot sleeve, 2) the runner system, and 3) the mold cavity. Note, even if the turbulence exists in some regions, it doesn't necessarily mean that all the flow field is turbulent.

[10] The problems in these experiments were, among other things, no simulation of the dimensional numbers such as Re, Geometry etc. and therefore different differential equations not typical to die casting were "solved." ??punctuation inside quotes] The researchers also look at what is known as a "poor design" for disturbances to flow downstream (this is like putting screen in the flow). However, a good design requires smooth contours.

[11] Beside other problems such as different flow velocity in different gates which were never really measured, the pressure in the cavity and quality of the liquid metal entering the cavity (is it in two phase?) were never recorded.

Transition from laminar to turbulent

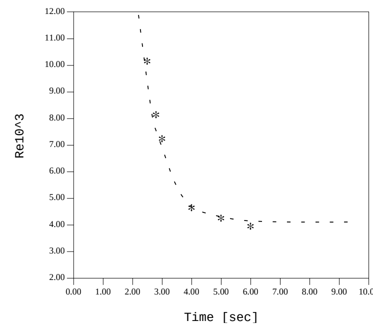


Fig. -3.9. Transition to turbulent flow in circular pipe for instantaneous flow after Wygnanski and others by interpolation.

Is the flow in the shot sleeve turbulent as the EKK sale engineers claim? These sale engineers did not present any evidence or analysis for such claims. For a simple analysis, the initial part of the shot sleeve filling, the liquid metal goes through a hydraulic jump. The flow after the hydraulic jump is very slow because the increase of the ratio of cross section areas. For example, casting of the 1[kg] from height of 0.2[m] to a shot sleeve of 0.1[m] creates a velocity in shot sleeve of $\sqrt{2}[m/sec]$ which results after the hydraulic jump to be with velocity about 0.01[m/sec]. The Reynolds number for this velocity is $\sim 10^4$ and Froude number of about 10. After the jump the Froude number is reduced and the flow is turbulent. However, by the time the hydraulic jump vanishes, the flow turns into laminar flow and no change (waviness) in the surface can be observed. It can be noticed that the time scale for the dissipation is about the same scale as the time for the operation of the next stage.

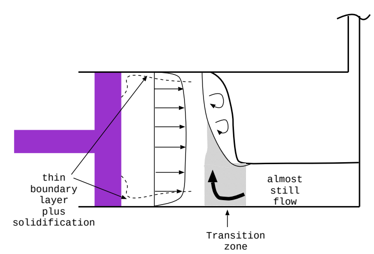


Fig. -3.10. Flow pattern in the shot sleeve.

Figure 3.9 exhibits the transition to a turbulent flow for instantaneous starting flow in a circular pipe. The abscissa represents time and the y–axis represents the Re number at which transition to turbulence occurs. The points on the graphs show the transition to a turbulence. This figure demonstrates that a large time is required to turn the flow pattern to turbulent which is measured in several seconds. The figure demonstrates that the transition does not occur below a certain critical Re number (known as the critical Re number for steady state). It also shows that a considerable time has elapsed before transition to turbulence occurs even for a relatively large Reynolds number. The geometry in die casting however is different and therefore it is expected that the transition occurs at different times. Our present knowledge of this area is very limited. Yet, a similar transition delay is expected to occur after the "instantaneous" start–up which probably will be measured in seconds. The flow in die casting in many situations is very short (in order of milliseconds) and therefore it is expected that the transition to a turbulent flow does not occur.

After the liquid metal is poured, it is normally repose for sometime in a range of 10 seconds. This fact is known in the scientific literature as the quieting time for which the existed turbulence (if exist) is reduced and after enough time (measured in seconds) is illuminated. Hence, the turbulence, which was created during the filling process of the shot sleeve, 'disappear" due to viscous dissipation. The question is, whether the flow in the duration of the slow plunger velocity turbulent (see Figure 3.10) can be examined.

Clearly, the flow in the substrate (a head of the wave) is still (almost zero velocity) and therefore the turbulence does not exist. The Re number behind the wave is above the critical Re number (which is in the range of 2000–3000). The typical time for the wave to travel to the end of the shot sleeve is in the range of a $\sim 10^0$ second. At present there are no experiments on the flow behind the wave[12]. The estimation can be done by looking at what is known in the literature about the transition to turbulence in instantaneous starting pipe flow. It has been shown [32] that the flow changes from laminar flow to turbulent flow in an abrupt manner for a flow with supercritical Re number.

A typical velocity of the propagating front (transition between laminar to turbulent) is about the same velocity as the mean velocity of the flow. Hence, it is reasonable to assume that the turbulence is confined to a small zone in the wave front since the wave is traveling in a faster velocity than the mean velocity. Note that the thickness of the transition layer is a monotone increase function of time (traveling distance). The Re number in the shot sleeve based on the diameter is in a range of $\sim 10^4$ which means that the boundary layer has not developed much. Therefore, the flow can be assumed as almost a plug flow with the exception of the front region.

A Note on Numerical Simulations

The most common model for turbulence that is used in the die casting industry for simulating the flow in cavity is $k - -\epsilon$. This model is based on several assumptions

1. isentropic homogeneous turbulence,
2. constant material properties (or a mild change of the properties),
3. continuous medium (only liquid (or gas), no mixing of the gas, liquid and solid whatsoever), and
4. the dissipation does not play a significant role (transition to laminar flow).

The $k - -\epsilon$ model is considered reasonable for the cases where these assumptions are not far from reality. It has been shown, and should be expected, that in cases where assumptions are far from reality, the $k - -\epsilon$ model produces erroneous results. Clearly, if we cannot determine whether the flow is turbulent and in what zone, the assumption of isentropic homogeneous turbulence is very questionable. Furthermore, if the change to turbulence just occurred, one cannot expect the turbulence to have sufficient time to become isentropic homogeneous. As if this is not enough complication, consider the effects of properties variations as a result of temperature change. Large variations of the properties such as the viscosity have been observed in many alloys especially in the mushy zone.

[12] It has to be said that similar situations are found in two phase flow but they are different by the fact the flow in two phase flow is a sinusoidal in some respects.

While the assumption of the continuous medium is semi reasonable in the shot sleeve and runner, it is far from reality in the die cavity. As discussed previously, the flow is atomized and it is expected to have a large fraction of the air in the liquid metal and conversely some liquid metal drops in the air/gas phase. In such cases, the isentropic homogeneous assumption is very dubious.

For these reasons the assumption of $k--\epsilon$ model seems unreasonable unless good experiments can show that the choice of the turbulence model does not matter in the calculation.

The question whether the flow in die cavity is turbulent or laminar is secondary. Since the two phase flow effects have to be considered such as atomization, air/gas entrainment etc. to describe the real flow in the cavity.

Additional note on numerical simulation

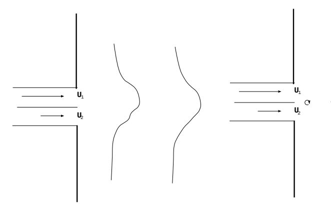


Fig. -3.11. Two streams of fluids into a medium.

The solution of momentum equation for certain situations may lead to unstable solution. Such case is the case of two jets with different velocity flow into a medium and they are adjoined (see Figure 3.11). The solution of such flow can show that the velocity field can be an unstable solution for which the flow moderately changes to become like wave flow. However, in many cases this flow can turn out to be full with vortexes and such. The reason that this happened is the introduction of instabilities. Numerical calculations intrinsically are introducing instabilities because of truncation of the calculations. In many cases, these truncations results in over–shooting or under–shooting of the nature instability. In cases where the flow is unstable, a careful study is required to make sure that the solution did not produce an unrealistic solution for larger or smaller than reality introduced instabilities. An excellent example of such poor understating is a work made in EKK company [2]. In that work, the flow in the shot sleeve was analyzed. The nature of the flow is two dimensional which can be seen by all the photos taken by numerous people (staring from the 50s). The presenter of that work explained that they have used 3D calculations because they want to study the instabilities perpendicular to the flow direction. The numerical "instability" in this case is larger than real instabilities and therefore, the numerical results show phenomena does not exist in reality.

Reverse transition from turbulent flow to laminar flow

After filling the die cavity, during the solidification process and intensification, the attained turbulence (if exist) is reduced and probably eliminated, i.e. the flow is laminar in a large portion of the solidification process. At present we don't comprehend when the transition point/criteria occurs and we must resort to experiments. It is a hope that some real good experiments using the similarity technique, outlined in this book, will be performed. So more knowledge can be gained and hopefully will appear in this book.

3.3.2 Dissipation effect on the temperature rise

The large velocities of the liquid metal (particularly at the runner) theoretically can increase the liquid metal temperature. To study this phenomenon, compare the of maximum effect of all the kinetic energy that is transformed into thermal energy.

$$\frac{U^2}{2} = c_p \Delta T \tag{3.13}$$

This equation leads to the definition of Eckert number

$$Ec = \frac{U^2}{c_p \Delta T} \tag{3.14}$$

When Ec number is very large it means that the dissipation plays a significant role and conversely when Ec number is small the dissipation effects are minimal. In die casting, Eckert number, Ec, is very small therefore the thermal dissipation is very small and can be ignored.

3.3.3 Gravity effects

The gravity has a large effect only when the gravity force is large relatively to other forces. A typical velocity range generated by gravity is the same as for an object falling through the air. The air effects can be neglected since the air density is very small compared with liquid metal density. The momentum is the other dominate force in the filling of the cavity. Thus, the ratio of the momentum force to the gravity force, also known as Froude number, determines if the gravity effects are important. The Froude number is defined here as

$$Fr = \frac{U^2}{\ell g} \tag{3.15}$$

Where U is the velocity, ℓ is the characteristic length g is the gravity force. For example, the characteristic pouring length is in order of 0.1[m], in extreme cases the velocity can reach 1.6[m] with characteristic time of 0.1[sec]. The author is not aware of experiments to verify the flow pattern in such cases (low Pr number due to solidification effect)[13] Yet, it is reasonable to assume that the liquid metal in such a case, flow in laminar regimes even though the Re number is relatively large ($\sim 10^4$) because of the short time and the short distance. The Re number is defined by the flow rate and the thickness of the exiting typical dimension. Note, the velocity reached its maximum value just before impinging on the sleeve surface.

[13] It be interesting to find such experiments.

The gravity has dominate effects on the flow in the shot sleeve since the typical value of the Froude number in that case (especially during the slow plunger velocity period) is in the range of one(1). Clearly, any analysis of the flow has to take into consideration the gravity (see Chapter 8).

3.4 Estimates of the time scales in die casting

3.4.1 Utilizing semi dimensional analysis for characteristic time

The characteristic time scales determine the complexity of the problem. For example, if the time for heat transfer/solidification process in the die cavity is much larger than the filling time, then the problem can be broken into three separate cases 1) the fluid mechanics, the filling process, 2) the heat transfer and solidification, and 3) dissipation (maybe considered with solidification). Conversely, the real problem in die filling is that we would like for the heat transfer process to be slower than the filling process, to ensure a proper filling. The same can be said about the other processes.

filling time

The characteristic time for filling a die cavity is determined by

$$t_f \sim \frac{L}{U} \tag{3.16}$$

Where L denotes the characteristic length of the die and U denotes the average filling velocity, determined by the pQ^2 diagram, in most practical cases this time typically is in order of 5–100 [millisecond]. Note, this time is not the actual filling time but related to it.

Atomization time

The characteristic time for atomization for a low Re number (large viscosity) is given by

$$t_{a_{viscosity}} = \frac{\nu \ell}{\sigma} \tag{3.17}$$

where ν is the kinematic viscosity, σ is the surface tension, and ℓ is the thickness of the gate. The characteristic time for atomization for large Re number is given by

$$t_{a_{momentum}} = \frac{\rho \ell^2 U}{\sigma} \tag{3.18}$$

The results obtained from these equations are different and the actual atomization time in die casting has to be between these two values.

Conduction time (die mold)

The governing equation for the heat transfer for the die reads

$$\rho_d {c_p}_d \frac{\partial T_d}{\partial t} = k_d \left(\frac{\partial^2 T_d}{\partial x^2} + \frac{\partial^2 T_d}{\partial y^2} + \frac{\partial^2 T_d}{\partial z^2} \right) \tag{3.19}$$

To obtain the characteristic time we dimensionless–ed the governing equation and present it with a group of constants that determine value of the characteristic time by setting it to unity. Denoting the following variables as

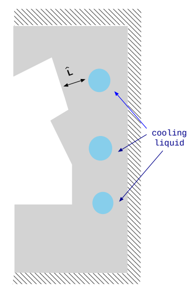


Fig. -3.12. Schematic of heat transfer processes in the die.

$$t_d' = \frac{t}{t_{c_d}} \qquad x_d' = \frac{x}{\tilde{L}}$$

$$y_d' = \frac{y}{\tilde{L}} \qquad z_d' = \frac{z}{\tilde{L}} \qquad \theta_d = \frac{T - T_B}{T_M - T_B} \tag{3.20}$$

$\tilde{L}$ the characteristic path of the heat transfer from the die inner surface to the cooling channels

subscript

B boiling temperature of cooling liquid

M liquid metal melting temperature

With these definitions, equation (3.19) is transformed to

$$\frac{\partial \theta_d}{\partial t} = \frac{t_{c_d} \alpha_d}{\tilde{L}^2} \left(\frac{\partial^2 \theta_d}{\partial x'^2} + \frac{\partial^2 \theta_d}{\partial y'^2} + \frac{\partial^2 \theta_d}{\partial z'^2} \right) \tag{3.21}$$

which leads into estimate of the characteristic time as

$$t_{c_d} \sim \frac{\tilde{L}^2}{\alpha_d} \tag{3.22}$$

Note the characteristic time is not effected by the definition of the θ_d.

Conduction time in the liquid metal (solid)

The governing heat equation in the solid phase of the liquid metal is the same as equation (3.19) with changing properties to liquid metal solid phase. The characteristic time for conduction is derived similarly as done previously by introducing the dimensional parameters

$$t' = \frac{t}{t_{c_s}};\ x' = \frac{x}{\ell};\ y' = \frac{y}{\ell};\ z' = \frac{z}{\ell};\ \ \theta_s = \frac{T - T_B}{T_M - T_B} \tag{3.23}$$

where t_{c_s} is the characteristic time for conduction process and, ℓ, denotes the main path of the heat conduction process die cavity. With these definitions, similarly as was done before the characteristic time is given by

$$t_{c_s} \sim \frac{\ell^2}{\alpha_s} \tag{3.24}$$

Note again that α_s has to be taken for properties of the liquid metal in the solid phase. Also note that the solidified length, ℓ, changes during the process and discussing the case where the whole die is solidified is not of interest. Initially the thickness, $\ell = 0$ (or very small). The characteristic time for very thin layers is very small, $t_{c_s} \sim 0$. As the solidified layer increases the characteristic time also increases. However, the temperature profile is almost established (if other processes were to remain in the same conditions). Similar situations can be found when a semi infinite slab undergoes solidification with ΔT changes as well as results of increase in the resistance. For the foregoing reasons the characteristic time is very small.

Solidification time

Miller's approach

Following Eckert's work, Miller and his student [20] altered the calculations[14] and based the assumption that the conduction heat transfer characteristic time in die (liquid metal in solid phase) is the same order magnitude as the solidification time. This assumption leads them to conclude that the main resistance to the solidification is in the interface between the die and mold [15]. Hence they conclude that the solidified front moves according to the following

$$\rho h_{sl} v_n = h \Delta T \tag{3.25}$$

Where here h is the innovative heat transfer coefficient between solid and solid[16] and v_n is front velocity. Then the filling time is given by the equation

$$t_s = \frac{\rho h_{sl}}{h \Delta T} \ell \tag{3.26}$$

[14] Miller and his student calculate the typical forces required for clamping. The calculations of Miller has shown an interesting phenomenon in which small casting (2$[kg]$) requires a larger force than heavier casting (20$[kg]$)?! Check it out in their paper, page 43 in NADCA Transaction 1997! If the results extrapolated (not to much) to about 50$[kg]$ casting, no force will be required for clamping. Furthermore, the force for 20 $[kg]$ casting was calculated to be in the range of $4000[N]$. In reality, this kind of casting will be made on 1000 [ton] machine or more (3 order of magnitude larger than Miller calculation suggested). The typical required force should be determined by the plunger force and the machine parts transient characteristics etc. Guess, who sponsored this research and how much it cost!

[15] An example how to do poor research. These kind of research works are found abundantly in Dr. Miller and Dr. J. Brevick from Ohio State Univerity. These works when examined show contractions with the logic and the rest of the world of established science.

[16] This coefficient is commonly used either between solid and liquid, or to represent the resistance between two solids. It is hoped that Miller and coworkers refer that this coefficient to represent the resistance between the two solids since it is a minor factor and does not determine the characteristic time.

where ℓ designates the half die thickness. As a corollary conclusion one can arrive from this construction is that the filling time is linearly proportional to the die thickness since $\rho h_{sl}/h\Delta T$ is essentially constant (according to Miller). This interesting conclusion contradicts all the previous research about solidification problem (also known as the Stefan problem). That is if h is zero the time is zero also. The author is not aware of any solidification problem to show similar results. Of course, Miller has all the experimental evidence to back it up!

Present approach

Heat balance at the liquid-solid interface yields

$$\rho_s h_{sf} v_n = k \frac{\partial (T_l - T_s)}{\partial n} \tag{3.27}$$

where n is the direction perpendicular to the surface and ρ has to be taken at the solid phase see Appendix 10. Additionally note that in many alloys, the density changes during the solidification and is substantial which has a significant effect on the moving of the liquid/solid front. It can be noticed that at the die interface $k_s \partial T/\partial n \cong k_d \partial T/\partial n$ (opposite to Miller) and further it can be assumed that temperature gradient in the liquid side, $\partial T/\partial n \sim 0$, is negligible compared to other fluxes. Hence, the speed of the solid/liquid front moves

$$v_n = \frac{k}{\rho_s h_{sl}} \frac{\partial T_s - T_l}{\partial n} \sim \frac{k \Delta T_{MB}}{\rho_s h_{sl} \tilde{L}} \tag{3.28}$$

Notice the difference to equation (3.26) The main resistance to the heat transfer from the die to the mold (cooling liquid) is in the die mold. Hence, the characteristic heat transfer from the mold is proportional to $\Delta T_{MB}/\tilde{L}$[17]. The characteristic temperature difference is between the melting temperature and the boiling temperature. The time scale for the front can be estimated by

$$t_s = \frac{\ell}{v_s} = \frac{\rho_s h_{sl} \ell^2 \left(\frac{\tilde{L}}{\ell}\right)}{k_d \Delta T_{MB}} \tag{3.29}$$

Note that the solidification time isn't a linear function of the die thickness, ℓ, but a function of $\sim \left(\ell^2\right)$[18].

Dissipation Time

Examples of how dissipation is governing the flow can be found abundantly in nature.

$$\left(\frac{\partial \theta_l}{\partial t} + u\frac{\partial \theta_l}{\partial x} + v\frac{\partial \theta_l}{\partial y} + w\frac{\partial \theta_l}{\partial z}\right) = \alpha_l \left(\frac{\partial^2 \theta_l}{\partial x^2} + \frac{\partial^2 \theta_l}{\partial y^2} + \frac{\partial^2 \theta_l}{\partial z^2}\right) + \mu \Phi \tag{3.30}$$

[17] The estimate can be improved by converting the resistances of the die to be represented by die length and the same for the other resistance into the cooling liquid i.e. $\Sigma 1/h_o + \tilde{L}/k + C_D ots + 1/h_i$.

[18] $\tilde{L}$ can be represented by ℓ for example, see more simplified assumption leads to pure $= \ell^2$.

Where Φ the dissipation function is defined as

$$\Phi = 2\left[\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial y}\right)^2 + \left(\frac{\partial v}{\partial y}\right)^2\right] + \left[\frac{\partial v}{\partial x}\right) + \left(\frac{\partial u}{\partial y}\right]^2 +$$
$$\left[\frac{\partial w}{\partial y}\right) + \left(\frac{\partial v}{\partial z}\right]^2 + \left[\frac{\partial w}{\partial y}\right) + \left(\frac{\partial v}{\partial z}\right]^2 - \frac{2}{3}\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right)^2 \tag{3.31}$$

Since the dissipation characteristic time isn't commonly studied in "regular" fluid mechanics, we first introduce two classical examples of dissipation problems. First problem deals with the oscillating manometer and second problem focuses on the "rigid body" brought to a rest in a thin cylinder.

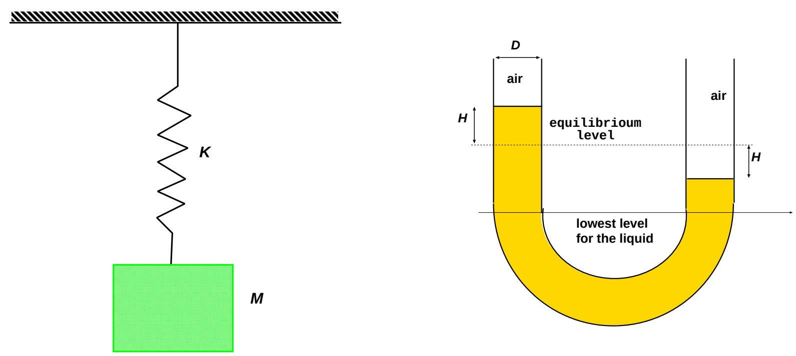


Fig b. *Oscillating manometer*

Fig a. *Mass, spring*

Fig. -3.13. The oscillating manometer for the example 3.1.

Example 3.1:

A liquid in manometer is disturbed from a rest by a distance of H_0. Assume that the flow is laminar and neglected secondary flows. Describe $H(t)$ as a function of time. Defined 3 cases: 1)under damping, 2) critical damping, and 3) over damping. Discuss the physical significance of the critical damping. Compute the critical radius to create the critical damping. For simplicity assume that liquid is incompressible and the velocity profile is parabolic.

Solution

The conservation of the mechanical energy can be written as

$$\frac{d}{dt}\begin{pmatrix}\text{rate of increase}\\ \text{of kinetic and}\\ \text{potential energy}\\ \text{in system}\end{pmatrix} = \Delta\begin{pmatrix}\text{total of inflow of}\\ \text{kinetic energy}\end{pmatrix} + \Delta\begin{pmatrix}\text{total of inflow of}\\ \text{potential energy}\end{pmatrix} +$$
$$\Delta\begin{pmatrix}\text{total of inflow of}\\ \text{potential energy}\end{pmatrix} + \Delta\begin{pmatrix}\text{total net rate of}\\ \text{surroundings}\\ \text{work on the}\\ \text{system}\end{pmatrix} + \Delta\begin{pmatrix}\text{total work due}\\ \text{to expansion or}\\ \text{compression of}\\ \text{fluid}\end{pmatrix} + \Delta\begin{pmatrix}\text{total rate}\\ \text{mechanical}\\ \text{energy}\\ \text{dissipated}\\ \text{because}\\ \text{viscosity}\end{pmatrix} \tag{3.32}$$

The chosen system is the liquid in the manometer. There is no flow in or out of the liquid of the manometer, and thus, terms that deal with flow in or out are canceled. It is assumed that the surface at the interface is straight without end effects like surface tension. This system is unsteady and therefore the velocity profile is function of the time and space. In order to demonstrate the way the energy dissipation is calculated it is assumed the velocity is function of the radius and time but separated. This assumption is wrong and cannot be used for real calculations because the real velocity profile is not separated and can have positive and negative velocities. It is common to assume that velocity profile is parabolic which is for the case where steady state is obtained.

This assumption can used as a limiting case and the velocity profile is

$$U(r,t) = U(r) = U_0(t)\left[1-\left(\frac{r}{R}\right)^2\right] \tag{3.33}$$

where R the radius of the manometer. The velocity at the center is a function of time but independent of the Length. It can be noticed that this equation dim:eq:velocityH is problematic because it breaks the assumption of the straight line of the interface.

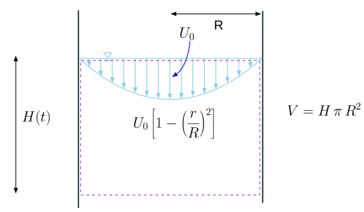


Fig. -3.14. Mass Balance to determine the relationship between the U_0 and the Height, H.

The relationship between the velocity at the center, U_0 to the height, $H(t)$ can be obtained from mass conservation on left side of the manometer (see Figure 3.14) is

$$\frac{d\left(\cancel{\rho}\,H\,\pi\,R^2\right)}{dt} = \int_0^R \cancel{\rho}\,U_0\left[1-\left(\frac{r}{R}\right)^2\right]\overbrace{2\,\pi\,r\,dr}^{dA} \tag{3.34}$$

Equation (3.34) relates $H(t)$ to the center velocity, U_0, and the integration results in

$$\frac{d\,H}{dt} = \frac{U_0}{2} \tag{3.35}$$

Note that $H(t)$ isn't a function of the radius, R. This relationship (3.35) is based on the definition that U_0 is positive for the liquid flowing to right and therefore the height decreases. The total kinetic energy in the tube is then

$$K_k = \int_0^L\int_0^R \frac{\rho\,{U_0}^2}{2}\left[1-\left(\frac{r}{R}\right)^2\right]^2 \overbrace{2\,\pi\,r\,dr}^{dA}\,d\ell = \frac{L\,{U_0}^2\,\pi\,R^2}{6} \tag{3.36}$$

where L is the total length (from one interface to another) and $d\ell$ is a coordinate running along the axis of the manometer neglecting the curvature of the "U" shape. It can be noticed that L is constant for incompressible flow. It can be observed that the disturbance of the manometer creates a potential energy which can be measured from a datum at the maximum lower point. The maximum potential energy is obtained when H is either maximum or minimum. The maximum kinetic energy is obtained when H

is zero. Thus, at maximum height, H_0 the velocity is zero. The total potential of the system is then

$$K_p = \overbrace{\int_0^{H_0-H} (\rho\, g\, \ell)\, \overbrace{\pi\, R^2 d\ell}^{dV}}^{\text{left side}} + \overbrace{\int_0^{H_0+H} (\rho\, g\, \ell)\, \overbrace{\pi\, R^2 d\ell}^{dV}}^{\text{right side}} = \left({H_0}^2 + H^2\right)\, \rho\, g\, \pi\, R^2 \tag{3.37}$$

The last term to be evaluated is the viscosity dissipation. Based on the assumptions in the example, the velocity profile is function only of the radius thus the only gradient of the velocity is in the r direction. Hence

$$E_d = \mu\, \Phi = L\, \mu \int_0^R \left(\frac{dU}{dr}\right)^2 \overbrace{2\, \pi\, r\, dr}^{dA} \tag{3.38}$$

The velocity derivative can be obtained by using equation (3.33) as

$$\frac{dU}{dr} = U_0 \left(\frac{-2\, r}{R^2}\right) \Longrightarrow \left(\frac{dU}{dr}\right)^2 = \left(\frac{4\, r^2\, {U_0}^2}{R^4}\right) \tag{3.39}$$

Substituting equation (3.39) into equation (3.37) reads

$$E_d = \mu\, 2\, \pi\, L \int_0^R \overbrace{\frac{4\, r^2\, {U_0}^2}{R^2}}^{\left(\frac{dU}{dr}\right)^2} \frac{r}{R} \frac{dr}{R} = 2\, \pi\, L\, \mu\, R^2\, {U_0}^2 \tag{3.40}$$

The work done on system is neglected by surroundings via the pressure at the two interfaces because the pressure is assumed to be identical. Equation (3.32) is transformed, in this case, into

$$\frac{d}{dt}\left(K_k + K_p\right) = -E_d \tag{3.41}$$

The kinetic energy derivative with respect to time (using equation (3.35)) is

$$\frac{d\, K_k}{dt} = \frac{d}{dt}\left(\frac{L\, {U_0}^2\, \pi\, R^2}{6}\right) = \frac{L\, \pi\, R^2}{6} 2\, U_0\, \frac{U_0}{dt} = \frac{4\, L\, \pi\, R^2}{3} \frac{d\, H}{dt} \frac{d^2\, H}{dt^2} \tag{3.42}$$

The potential energy derivative with respect to time is

$$\frac{d\, K_p}{dt} = \frac{d}{dt}\left[\left({H_0}^2 + H^2\right)\, \rho\, g\, \pi\, R^2\right] = 2\, H \frac{dH}{dt}\, \rho\, g\, \pi\, R^2 \tag{3.43}$$

Substituting equations (3.43), (3.42) and (3.38) into equation (3.41) results in

$$\frac{4\, L\, \pi\, R^2}{3} \frac{d\, H}{dt} \frac{d^2\, H}{dt^2} + 2\, H \frac{dH}{dt}\, \rho\, g\, \pi\, R^2 + 2\, \pi\, L\, \mu\, {U_0}^2 = 0 \tag{3.44}$$

Equation (3.44) can be simplified using the identity of (3.35) to be

$$\frac{d^2 H}{dt^2} + \frac{6\,\mu}{\rho\,R^2}\frac{dH}{dt} + \frac{3\,g}{2\,L} H = 0 \tag{3.45}$$

This equation is similar to the case mass tied to a spring with damping. This equation is similar to RLC circuit[19]. The common method is to assume that the solution of the form of $A\,e^{\xi\,t}$ where the value of A and ξ will be such determined from the equation. When substituting the "guessed" function into result that ξ having two possible solution which are

$$\xi = \frac{-\frac{6\,\mu}{\rho\,R^2} \pm \sqrt{\left(\frac{6\,\mu}{\rho\,R^2}\right)^2 - \frac{6\,g}{L}}}{2} \tag{3.46}$$

Thus, the solution is

$$\begin{array}{ccc} H = A\,e^{\xi_1\,t} + A\,e^{\xi_2\,t} & \Longrightarrow & \xi_1 \neq \xi_2 \\ H = A\,e^{\xi\,t} + A\,e^{\xi\,t} & \Longrightarrow & \xi_1 = \xi_2 = \xi \end{array} \tag{3.47}$$

The constant A_1 and A_2 are to be determined from the initial conditions. The value under the square root determine the kind of motion. If the value is positive then the system is over–damped and the liquid height will slowly move the equilibrium point. If the value in square is zero then the system is referred to as critically damped and height will move rapidly to the equilibrium point. If the value is the square root is negative then the solution becomes a combination of sinuous and cosines. In the last case the height will oscillate with decreasing size of the oscillation. The critical radius is then

$$R_c = \sqrt[4]{\frac{6\,\mu^2\,L}{g\,\rho^2}} \tag{3.48}$$

It can be observed that this analysis is only the lower limit since the velocity profile is much more complex. Thus, the dissipation is much more significant.

End Solution

Example 3.2:

A thin ($t/D \ll 1$) cylinder full with liquid is rotating in a velocity, ω. The rigid body is brought to a stop. Assuming no secondary flows (Bernard's cell, etc.), describe the flow as a function of time. Utilize the ratio $1 \gg t/D$.

$$\frac{d^2X}{dt^2} + \left(\frac{\mu}{\ell^2}\right)\frac{dX}{dt} + X = 0 \tag{3.49}$$

Discuss the case of rapid damping, and the case of the characteristic damping

[19] An electrical circuit consisting of a resistor (R), an inductor (L), and a capacitor (C), connected in series or in parallel.

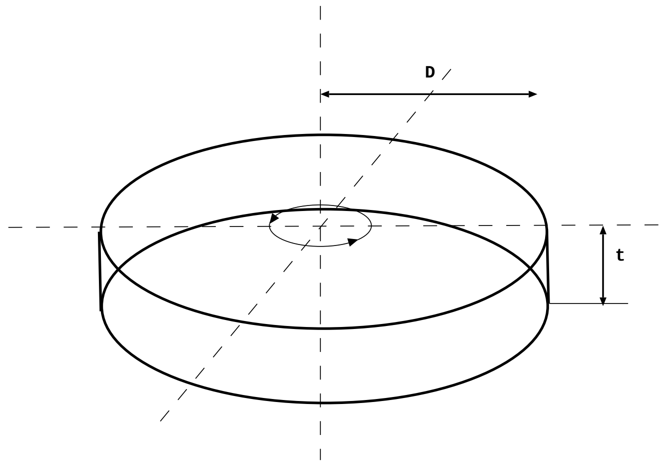


Fig. -3.15. Rigid body brought into rest.

Solution

End Solution

These examples illustrate that the characteristic time of dissipation can be assessed by $\sim \mu(du/d``y'')^2$ thus given by ℓ^2/ν. Note the analogy between t_s and t_{diss}, for which ℓ^2 appears in both of them, the characteristic length, ℓ, appears as the typical die thickness.

3.4.2 The ratios of various time scales

The ratio of several time ratios can be examined for typical die casting operations. The ratio of solidification time to the filling time

$$\frac{t_f}{t_s} \sim \frac{L k_d \Delta T_{MB}}{U \rho_s h_{sl} \ell \tilde{L}} = \frac{Ste}{Pr\ Re}\left(\frac{\rho_{lm}}{\rho_s}\right)\left(\frac{k_d}{k_{lm}}\right)\left(\frac{L}{\tilde{L}}\right) \tag{3.50}$$

where

Re Reynolds number $\frac{U\ell}{\nu_{lm}}$

Ste Stefan number $\frac{{c_p}_{lm}\Delta T_{MB}}{h_{sl}}$

the discussion is augmented on the importance of equation (3.50). The ratio is extremely important since it actually defines the required filling time.

$$t_f = C\left(\frac{\rho_{lm}}{\rho_s}\right)\left(\frac{k_d}{k_{lm}}\right)\left(\frac{L}{\tilde{L}}\right)\frac{Ste}{Pr\ Re} \tag{3.51}$$

At the moment, the "constant", C, is unknown and its value has to come out from experiments. Furthermore, the "constant" is not really a constant and is a very mild function of the geometry. Note that this equation is also different from all the previously proposed filling time equations, since it takes into account solidification and filling process[20].

The ratio of liquid metal conduction characteristic time to characteristic filling time is given by

$$\frac{t_{c_d}}{t_f} \sim \frac{U\tilde{L}^2}{L\alpha} = \frac{U\ell}{\nu}\,\frac{\nu}{\alpha}\,\frac{\tilde{L}^2}{L\ell} \;=\; Re\;Pr\,\frac{\tilde{L}^2}{L\ell} \tag{3.52}$$

The solidification characteristic time to conduction characteristic time is given by

$$\frac{t_s}{t_c} \sim \frac{\rho_s h_{sl}\ell\tilde{L}\alpha_d}{k_d \Delta T_{MB}\tilde{L}^2} = \frac{1}{Ste}\left(\frac{\rho_s}{\rho_d}\right)\left(\frac{c_{p_{lm}}}{c_{p_d}}\right)\left(\frac{\ell}{\tilde{L}}\right) \tag{3.53}$$

The ratio of the filling time and atomization is

$$\frac{t_{a_{viscosity}}}{t_f} \approx \frac{\nu\ell U}{\sigma L} = Ca\left(\frac{\ell}{L}\right) \sim 6\times\;10^{-8} \tag{3.54}$$

Note that ℓ, in this case, is the thickness of the gate and not of the die cavity.

$$\frac{t_{a_{momentum}}}{t_f} \approx \frac{\rho\ell^2U^2}{\sigma L} = We\left(\frac{\ell}{L}\right) \sim 0.184 \tag{3.55}$$

which means that if atomization occurs, it will be very fast compared to the filling process.

The ratio of the dissipation time to solidification time is given by

$$\frac{t_{diss}}{t_s} \sim \frac{\ell^2}{\nu_{lm}}\;\frac{k_d\Delta T_{MB}}{\rho_s h_{sl}\ell\tilde{L}} = \left(\frac{Ste}{Pr}\right)\left(\frac{k_d}{k_{lm}}\right)\left(\frac{\rho_{lm}}{\rho_s}\right)\left(\frac{\ell}{\tilde{L}}\right) \sim 10^0 \tag{3.56}$$

this equation yields typical values for many situations in the range of 10^0 indicating that the solidification process is as fast as the dissipation. It has to be noted that when the solidification progress, the die thickness decreases. The ratio, $\ell/\tilde{L}$, reduced as well. As a result, the last stage of the solidification can be considered as a pure conduction problem as was done by the "English" group.

3.5 *Similarity applied to Die cavity*

This section is useful for those who are dealing with research on die casting and or other casting process.

[20] In this book, this equation because of its importance is referred to as Eckert–BarMeir's equation. If you have good experimental work, your name can be added to this equation.

3.5.1 Governing equations

The filling of the mold cavity can be divided into two periods. In the first period (only fluid mechanics; minimum heat transfer/solidification) and the second period in which the solidification and dissipation occur. This discussion deals with how to conduct experiments in die casting[21]. It has to be stressed that the conditions down–stream have to be understood prior to the experiment with the die filling. The liquid metal velocity profile and flow pattern are still poorly understood at this stage. However, in this discussion we will assume that they are known or understood to same degree[22].

The governing equations are given in the preceding sections and now the boundary conditions will be discussed. The boundary condition at the solid interface for the gas/air and for the liquid metal are assumed to be "no–slip" condition which reads

$$u_g = v_g = w_g = u_{lm} = v_{lm} = w_{lm} = 0 \tag{3.57}$$

where the subscript g is used to indicate the gas phase. It is noteworthy to mention that this can be applied to the case where liquid metal is mixed with air/gas and both are touching the surface. At the interface between the liquid metal and gas/air, the pressure jump is expressed as

$$\frac{\sigma}{r_1 + r_2} \approx \Delta p \tag{3.58}$$

where r_1 and r_2 are the principal radii of the free surface curvature, and, σ, is the surface tension between the gas and the liquid metal. The surface geometry is determined by several factors which include the liquid movement[23] instabilities etc.

Now on the difficult parts, the velocity at gate has to be determined from the pQ^2 diagram or previous studies on the runner and shot sleeve. The difficulties arise due to the fact that we cannot assign a specific constant velocity and assume only liquid flow out. It has to be realized that due to the mixing processes in the shot sleeve and the runner (especially in a poor design process and runner system, now commonly used in the industry), some portions at the beginning of the process have a significant part which contains air/gas. There are several possibilities that the conditions can be prescribed. The first possibility is to describe the pressure variation at the entrance. The second possibility is to describe the velocity variation (as a function of time). The velocity is reduced during the filling of the cavity and is a function of the cavity geometry. The change in the velocity is sharp in the initial part of the filling due to the change from a free jet to an immersed jet. The pressure varies also at the entrance, however, the variations are more mild. Thus, it is a better possibility[24] to consider the pressure prescription. The simplest assumption is constant pressure

$$P = P_0 = \frac{1}{2}\rho {U_0}^2 \tag{3.59}$$

[21] Only minimal time and efforts was provided how to conduct experiments on the filling of the die. In the future, other zones and different processes will be discussed.

[22] Again the die casting process is a parabolic process.

[23] Note, the liquid surface cannot be straight, for unsteady state, because it results in no pressure gradient and therefore no movement.

[24] At this only an intelligent guess is possible.

We also assume that the air/gas obeys the ideal gas model.

$$\rho_g = \frac{P}{RT} \tag{3.60}$$

where R is the air/gas constant and T is gas/air temperature. The previous assumption of negligible heat transfer must be inserted and further it has to be assumed that the process is polytropic[25]. The dimensionless gas density is defined as

$$\rho' = \frac{\rho}{\rho_0} = \left(\frac{P_0}{P}\right)^{\frac{1}{n}} \tag{3.61}$$

The subscript 0 denotes the atmospheric condition.

The air/gas flow rate out the cavity is assumed to behave according to the model in Chapter 9. Thus, the knowledge of the vent relative area and $\frac{4fL}{D}$ are important parameters. For cases where the vent is well designed (vent area is near the critical area or above the density, ρ_g can be determined as was done by [5]).

To study the controlling parameters, the equations are dimensionless–ed. The mass conservation for the liquid metal becomes

$$\frac{\partial \rho_{lm}}{\partial t'} + \frac{\partial \rho_{lm}\, {u'}_{lm}}{\partial x'} + \frac{\partial \rho_{lm}\, {v'}_{lm}}{\partial y'} + \frac{\partial \rho_{lm}\, {w'}_{lm}}{\partial z'} = 0 \tag{3.62}$$

where $x' = \frac{x}{\ell}$, $y' = y/\ell$, $z' = z/\ell$, $u' = u/U_0$, $v' = v/U_0$, $w' = w/U_0$ and the dimensionless time is defined as $t' = \frac{tU_0}{\ell}$, where $U_0 = \sqrt{2P_0/\rho}$.

Equation (3.62) can be similar under the assumption of constant density to read

$$\frac{\partial {u'}_{lm}}{\partial x'} + \frac{\partial {v'}_{lm}}{\partial y'} + \frac{\partial {w'}_{lm}}{\partial z'} = 0 \tag{3.63}$$

Please note that this simplification can be used for the gas phase. The momentum equation for the liquid metal in the x-coordinate assuming constant density and no body forces reads

$$\frac{\partial \rho_{lm} {u'}_{lm}}{\partial t'} + u' \frac{\partial \rho_{lm} {u'}_{lm}}{\partial x'} + v' \frac{\partial \rho_{lm} {u'}_{lm}}{\partial y'} + w' \frac{\partial \rho_{lm} {u'}_{lm}}{\partial z'} = \\ -\frac{\partial {p'}_{lm}}{\partial x'} + \frac{1}{Re}\left(\frac{\partial^2 u'_{lm}}{\partial {x'}^2} + \frac{\partial^2 v'}{\partial {y'_{lm}}^2} + \frac{\partial^2 w'}{\partial {z'_{lm}}^2}\right) \tag{3.64}$$

where $Re = U_0 \ell / \nu_{lm}$ and $p' = p/P_0$.

The gas phase continuity equation reads

$$\frac{\partial {\rho'}_g}{\partial t'} + \frac{\partial \rho'_g\, u'_g}{\partial x'} + \frac{\partial \rho'_g\, v'_g}{\partial y'} + \frac{\partial \rho'_g\, w'_g}{\partial z'} = 0 \tag{3.65}$$

[25]There are several possibilities, this option is chosen only to obtain the main controlling parameters.

The gas/air momentum equation[26] is transformed into

$$\frac{\partial \rho'_g {u'}_g}{\partial t'} + u' \frac{\partial \rho'_g {u'}_g}{\partial x'} + v' \frac{\partial \rho'_g {u'}_g}{\partial y'} + w' \frac{\partial \rho'_g {u'}_g}{\partial z'} = -\frac{\partial {p'}_g}{\partial x'} + \underbrace{\frac{\nu_{lm}}{\nu_g} \frac{\rho_{g0}}{\rho_{lm}} \frac{1}{Re} \left(\frac{\partial^2 u'_g}{\partial {x'}^2} + \frac{\partial^2 v'_g}{\partial {y'}^2} + \frac{\partial^2 w'_g}{\partial {z'}^2} \right)}_{\sim 0} \tag{3.66}$$

Note that in this equation, additional terms were added, $(\nu_{lm}/\nu_g)(\rho_{g0}/\rho_{lm})$.

The "no-slip" conditions are converted to:

$$u'_g = v'_g = w'_g = u'_{lm} = v'_{lm} = w'_{lm} = 0 \tag{3.67}$$

The surface between the liquid metal and the air satisfy

$$p' \left(r'_1 + r'_2 \right) = \frac{1}{We} \tag{3.68}$$

where the p', r'_1, and r'_2 are defined as $r'_1 = r_1/\ell$ $r'_2 = r_2/\ell$

The solution to equations has the form of

$$\begin{aligned}
u' &= f_u \left(x', y', z', Re, We, \frac{A}{A_c}, \tfrac{4fL}{D}, n, \frac{\rho_g}{\rho_{lm}}, \frac{\nu_{lm}}{\nu_g} \right) \\
v' &= f_v \left(x', y', z', Re, We, \frac{A}{A_c}, \tfrac{4fL}{D}, n, \frac{\rho_g}{\rho_{lm}}, \frac{\nu_{lm}}{\nu_g} \right) \\
w' &= f_w \left(x', y', z', Re, We, \frac{A}{A_c}, \tfrac{4fL}{D}, n, \frac{\rho_g}{\rho_{lm}}, \frac{\nu_{lm}}{\nu_g} \right) \\
p' &= f_p \left(x', y', z', Re, We, \frac{A}{A_c}, \tfrac{4fL}{D}, n, \frac{\rho_g}{\rho_{lm}}, \frac{\nu_{lm}}{\nu_g} \right)
\end{aligned} \tag{3.69}$$

If it will be found that equation (3.66) can be approximated[27] by

$$\frac{\partial {u'}_g}{\partial t'} + u' \frac{\partial {u'}_g}{\partial x'} + v' \frac{\partial {u'}_g}{\partial y'} + w' \frac{\partial {u'}_g}{\partial z'} \approx -\frac{\partial {p'}_g}{\partial x'} \tag{3.70}$$

then the solution is reduced to

$$\begin{aligned}
u' &= f_u \left(x', y', z', Re, We, \frac{A}{A_c}, \tfrac{4fL}{D}, n \right) \\
v' &= f_v \left(x', y', z', Re, We, \frac{A}{A_c}, \tfrac{4fL}{D}, n \right) \\
w' &= f_w \left(x', y', z', Re, We, \frac{A}{A_c}, \tfrac{4fL}{D}, n \right) \\
p' &= f_p \left(x', y', z', Re, We, \frac{A}{A_c}, \tfrac{4fL}{D}, n \right)
\end{aligned} \tag{3.71}$$

[26] In writing this equation, it is assumed that viscosity of the air is independent of pressure and temperature.

[27] This topic is controversial in the area of two phase flow.

At this stage, it is not known if it is the case and if it has to come out from the experiments. The density ratio can play a role because two phase flow characteristic is a major part of the filling process.

3.5.2 Design of Experiments

Under Construction [28].

3.6 Summary of dimensionless numbers

This section summarizes all the major dimensionless parameters and what effects they have on the die casting process.

Reynolds number

$$Re = \frac{\rho U^2/\ell}{\nu U/\ell^2} = \frac{\text{internal Forces}}{\text{viscous forces}}$$

Reynolds number represents the ratio of the momentum forces to the viscous forces. In die casting, Reynolds number plays a significant role which determines the flow pattern in the runner and the vent system. The discharge coefficient, C_D, is used in the pQ^2 diagram is determined largely by the Re number through the value of friction coefficient, f, inside the runner.

Eckert number

$$Ec = \frac{1/2\rho U^2}{1/2\rho c_p \Delta T} = \frac{\text{inertial energy}}{\text{thermal energy}}$$

Eckert number determines if the role of the momentum energy transferred to thermal energy is significant.

Brinkman number

$$Br = \frac{\mu U^2/\ell^2}{k\Delta T/\ell^2} = \frac{\text{heat production by viscous dissipation}}{\text{heat transfer transport by conduction}}$$

Brinkman number is a measure of the importance of the viscous heating relative the conductive heat transfer. This number is important in cases where large velocity change occurs over short distances such as lubricant flow (perhaps, the flow in the gate). In die casting, this number has small values indicating that practically the viscous heating is not important.

[28] See for time being Eckert's paper

Mach number

$$Ma = \frac{U}{\sqrt{\frac{\gamma \partial p}{\partial \rho}}}$$

For ideal gas (good assumption for the mixture of the gas leaving the cavity). It becomes

$$M \cong \frac{U}{\sqrt{\gamma R T}} = \frac{\text{characteristic velocity}}{\text{gas sound velocity}}$$

Mach number determines the characteristic of flow in the vent system where the air/gas velocity is reaching to the speed of sound. The air is chocked at the vent exit and in some cases other locations as well for vacuum venting. In atmospheric venting the flow is not chocked for large portion of the process. Moreover, the flow, in well design vent system, is not chocked. Yet the air velocity is large enough so that the Mach number has to be taken into account for reasonable calculation of the C_D.

Ozer number

$$Oz = \frac{\frac{{C_D}^2 P_{max}}{\rho}}{\left(\frac{Q_{max}}{A_3}\right)^2} = \left(\frac{A_3}{Q_{max}}\right)^2 {C_D}^2 \frac{P_{max}}{\rho} = \frac{\text{effective static pressure energy}}{\text{average kinematic energy}}$$

One of the most important number in the pQ^2 diagram calculation is Ozer number. This number represents how good the runner is designed.

Froude number

$$Fr = \frac{\rho U^2/\ell}{\rho g} = \frac{\text{inertial forces}}{\text{gravity forces}}$$

Fr number represent the ratio of the gravity forces to the momentum forces. It is very important in determining the critical slow plunger velocity. This number is determined by the height of the liquid metal in the shot sleeve. The Froude number does not play a significant role in the filling of the cavity.

Capillary number

$$Ca = \frac{\rho U^2/\ell}{\rho g} = \frac{\text{inertial forces}}{\text{gravity forces}}$$

capillary number (Ca) determine when the flow during the filling of the cavity is atomized or is continuous flow (for relatively low Re number).

Weber number

$$We = \frac{1/2\rho U^2}{1/2\sigma/\ell} = \frac{\text{inertial forces}}{\text{surface forces}}$$

We number is the other parameter that govern the flow pattern in the die. The flow in die casting is atomized and, therefore, We with combinations of the gate design also determine the drops sizes and distribution.

Critical vent area

$$A_c = \frac{V(0)}{ct_{max} m_{max}}$$

The critical area is the area for which the air/gas is well vented.

3.7 Summary

The dimensional analysis demonstrates that the fluid mechanics process, such as the filling of the cavity with liquid metal and evacuation/extraction of the air from the mold, can be dealt with when heat transfer is neglected. This provides an excellent opportunity for simple models to predict many parameters in the die casting process. It is recommended for interested readers to read Eckert's book "Analysis of Heat and Mass transfer" to have better and more general understanding of this topic.

3.8 Questions

Under construction

CHAPTER 4

Fundamentals of Pipe Flow

Chapter Under heavy construction

4.1 Introduction

The die casting engineer encounters many aspects of network flow. For example, the liquid metal flows in the runner is a network flow. The flow of the air and other gases out of the mold through the vent system is also another example network flow. The pQ^2 diagram also requires intimate knowledge of the network flow. However, most die casting engineers/researchers are unfamiliar with fluid mechanics and furthermore have a limited knowledge and understanding of the network flow. Therefore, this chapter is dedicated to describe a brief introduction to a flow in a network. It is assumed that the reader does not have extensive background in fluid mechanics. However, it is assumed that the reader is familiar with the basic concepts such as pressure and force, work, power. More comprehensive coverage can be found in books dedicated to fluid mechanics and pipe flow (network for pipe). First a discussion on the relevancy of the data found for other liquids to the die casting process is presented. Later a simple flow in a straight pipe/conduit is analyzed. Different components which can appear in network are discussed. Lastly, connection of the components in series and parallel are presented.

4.2 Universality of the loss coefficients

Die casting engineers who are not familiar with fluid mechanics ask whether the loss coefficients obtained for other liquids should/could be used for the liquid metal.

To answer this question, many experiments have been carried out for different liquids flowing in different components in the last 300 years. An example of such exper-

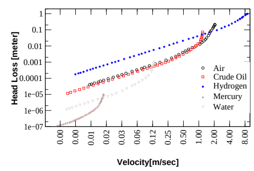


Fig a. *Friction of orifice as a function velocity.*

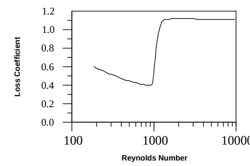


Fig b. *The collapsed results as funciton Reynolds number.*

Fig. -4.1. The results for the flow in a pipe with orifice.

iments is a flow of different liquids in a pipe with an orifice (see Figure 4.1). Different liquids create significant head loss for the same velocity. Moreover, the differences for the different liquids are so significant that the similarity is unclear as shown in Figure **??**. As the results of the past geniuses work, it can be shown that when results are normalized by Reynolds number (Re) instated of the velocity and when the head loss is replaced by the loss coefficient, $\frac{\Delta H}{U^2/2\,g}$ one obtains that all the lines are collapsed on to a single line as shown in Figure 4.1b. This result indicates that the experimental results obtained for one liquid can be used for another liquid metal provided the other liquid is a Newtonian liquid[1]. Researchers shown that the liquid metal behaves as Newtonian liquid if the temperature is above the mushy zone temperature. This example is not correct only for this spesific geometry but is correct for all the cases where the results are collapsed into a single line. The parameters which control the problem are found when the results are "collapsed" into a single line. It was found that the resistance to the flow for many components can be calculated (or extracted from experimental data) by knowing the Re number and the geometry of the component. In a way you can think about it as a prof of the dimensional analysis (presented in Chapter 3).

4.3 A simple flow in a straight conduit

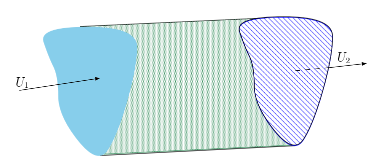


Fig. -4.2. General simple conduit description.

A simple and most common component is a straight conduit as shown in Figure 4.2. The simplest conduit is a circular pipe which would be studied here first. The entrance problem and the unsteady aspects will be discussed later. The parameters that the die casting engineers interested are the liquid metal velocity, the power to drive this velocity, and the pressure difference occur for the required/desired velocity. What determine these parameters? The velocity is determined by the pressure difference applied on the pipe and the resistance

[1] Newtonian liquid obeys the following stress law $\tau = \mu \frac{dU}{dy}$

to the flow. The relationship between the pressure difference, the flow rate and the resistance to the flow is given by the experimental equation (4.1). This equation is used because it works[2]. The pressure difference determined by the geometrical parameters and the experimental data which expressed by f[3] which can be obtained from Moody's diagram.

$$\Delta P = f \rho \frac{L}{D} \frac{U^2}{2}; \Delta H = f \frac{L}{D} \frac{U^2}{2\,g} \tag{4.1}$$

Note, head is energy per unit weight of fluid (i.e. Force x Length/Weight = Length) and it has units of length. Thus, the relationship between the Head (loss) and the pressure (loss) is

$$\Delta P = \frac{\Delta H}{\rho\, g} \tag{4.2}$$

The resistance coefficient for circular conduit can be defined as

$$K_F = f \frac{L}{D} \tag{4.3}$$

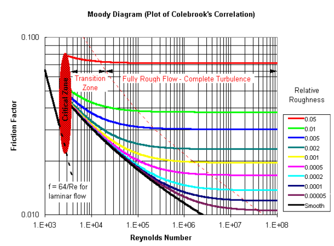


Fig. -4.3. General simple conduit description.

This equation is written for a constant density flow and a constant cross section. The flow rate is expressed as

$$Q = U\,A \tag{4.4}$$

The cross sectional area of circular is $A = \pi r^2 = \pi D^2/4$, using equation (4.4) and substituting it into equation (4.1) yields

$$\Delta P = f \rho \frac{16\,L}{\pi^2\,D^3}\,Q^2 \tag{4.5}$$

The equation (4.5) shows that the required pressure difference, ΔP, is a function of $1/D^3$ which demonstrates the tremendous effect the diameter has on the flow rate. The length, on the other hand, has mush less significant effect on the flow rate.

The power which requires to drive this flow is give by

$$\mathcal{P} = \mathcal{Q} \cdot \mathcal{P} \tag{4.6}$$

These equations are very important in the understanding the economy of runner design, and will be studied in Chapter 12 in more details.

The power in terms of the geometrical parameters and the flow rate is given

$$\mathcal{P} = \{\rho \frac{\infty\,\mathcal{L}}{\pi^{\in}\,\mathcal{D}^{\ni}}\,\mathcal{Q}^{\ni} \tag{4.7}$$

[2]Actually there are more reasons but they are out of the scope of this book

[3]At this stage, we use different definition than one used in Chapter A. The difference is by a factor of 4. Eventially we will adapt one system for the book.

4.3.1 Examples of the calculations

Example 4.1:

calculate the pressure loss (difference) for a circular cross section pipe for driving aluminum liquid metal at velocity of 10[m/sec] for a pipe length of 0.5 [m] (like a medium quality runner) with diameter of 5[mm] 10[mm] and 15[mm]

Solution

This is example 4.3.1

Example 4.2:

calculate the power required for the above example

Solution

4.4 Typical Components in the Runner andVent Systems

In the calculations of the runner the die casting engineer encounter beside the straight pipe which was dealt in the previous section but other kind of components. These components include the bend, Y–connection and tangential gate, "regular gate", the extended Y connection and expansion/contraction (including the abrupt expansion/contraction). In this section a general discussion on the good design practice for the different component is presented. A separate chapter is dedicated to the tangential runner due to its complication.

4.4.1 bend

The resistance in the bend is created because a change in the momentum and the flow pattern. Engineers normally convert the bend to equivalent conduit length. This conversion produces adequate results in same cases while in other it might introduce larger error. The knowledge of this accuracy of this conversion is very limited because limited study have been carry out for the characteristic of flows in die casting. From the limited information the author of this book gadered it seem that it is reasonable to carry this conversion for the calculations of liquid metal flow resistance while in the air/liquid metal mixture it far from adequate. Moreover, "hole" of our knowledge of the gas flow in vent system are far more large. Nevertheless, for the engineering purpose at this stage it seem that some of the errors will cancel each other and the end result will be much better.

bad english, change it please

The schematic of a bend commonly used in die casting is shown in Figure **??**. The resistance of the bend is a function of several parameters: angle, θ, radius, R and the geometry before and after the

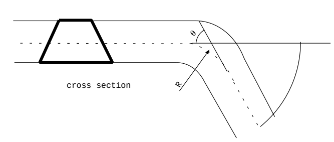


Fig. -4.4. A sketch of the bend in die casting.

bend. Commonly, the runner is made with the same geometry before and after the bend. Moreover, we will assume in this discussion that downstream and upstream do not influence that flow in the flow. This assumption is valid when there is no other bend or other change in the flow nearby. In cases that such a change(s) exists more complicated analysis is required.

In the light of the for going discussion, we left with two parameters that control the resistance, the angle, θ, and the radius, R As larger the angle is larger the resistance will be. In the practice today, probably because the way the North American Die Casting Association teaching, excessive angle can be found through the industry. It is recommended never to exceed the straight angle (90^0). Figure **??** made from a data taken from several sources. From the Figure it is clear that optimum radius should be around 3.

4.4.2 Y connection

picture of Y connection

The Y–connection reprsent a split in the runner system. The resistence

4.4.3 Expansion/Contraction

One of the undisirable element is the runner system is sudden change in the conduict area. In some instance they are inevodeble. We will disscuss how to design and what are the better design options which availble for the engineer.

4.5 *Putting it all to Together*

There are two main kinds of connections; series and parallel. The resistance in the series connection has to be added in a fashion similar to electrical resistance i.e. every resistance has to be added plainly to the total resistance. There are many things that contribute to the resistance besides the regular length, i.e. bends, expansions, contractions etc. All these connections are of series type.

4.5.1 Series Connection

The flow rate in different locations is a function of the temperature. Eckert [13] demonstrated that the heat transfer is insignificant in the duration of the filling of the cavity, and therefore the temperature of the liquid metal can be assumed almost constant during the filling period (which in most cases is much less 100 milliseconds). As such, the solidification is insignificant (the liquid metal density changes less than 0.1% in the runner); therefore, the volumetric flow rate can be assumed constant:

$$Q_1 = Q_2 = Q_3 = Q_i \tag{4.8}$$

Clearly, the pressure in the points is different and

$$P_1 \neq P_2 \neq P_3 \neq \Pi \tag{4.9}$$

However the total pressure loss is composed of from all the small pressure loss

$$P_1 - P_{end} = (P_1 - P_2) + (P_2 - P_3) + \cdots \tag{4.10}$$

Every single pressure loss can be written as

$$P_{i-1} - P_i = K_i \frac{U^2}{2} \tag{4.11}$$

There is also resistance due to parallel connection i.e. y connections, y splits and manifolds etc. First, lets look at the series connection. (see Figure **??**). where:

K_{bend}	the resistance in the bend
L	length of the duct (vent),
f	friction factor, and

4.5.2 Parallel Connection

An example of the resistance of parallel connection (see Figure **??**).

The pressure at point 1 is the same for two branches however the total flow rate is the combination

$$Q_{total} = Q_i + Q_j \tag{4.12}$$

between two branches and the loss in the junction is calculated as

To add a figure and check if the old one is good

Fig. -4.5. A parallel connection

CHAPTER 5

Flow in Open Channels

5.1 Introduction

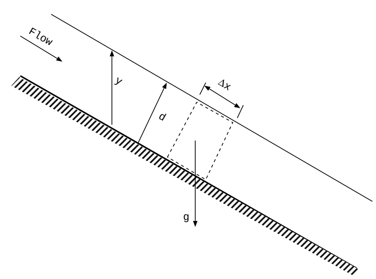


Fig. -5.1. Equilibrium of Forces in an open channel.

One of the branches of the fluid mechanics discussed in Chapter 2. Here we expand this issue further because it is give the basic understanding to the "wave" phenomenon. There are numerous books that dealing with open channel flow and the interested reader can broader his/her knowledge by reading book such as Open-Channel Hydraulics by Ven Te Chow (New York: McGraw-Hill Book Company, Inc. 1959). Here a basic concepts for the non-Fluid Mechanics Engineers are given.

The flow in open channel flow in steady state is balanced by between the gravity forces and mostly by the friction at the channel bed. As one might expect, the friction factor for open channel flow has similar behavior to to one of the pipe flow with transition from laminar flow to the turbulent at about $Re \sim 10^3$. Nevertheless, the open channel flow has several respects the cross section are variable, the surface is at almost constant pressure and the gravity force are important.

The flow of a liquid in a channel can be characterized by the specific energy that is associated with it. This specific energy is comprised of two components: the hydrostatic pressure and the liquid velocity[1].

[1]The velocity is an average velocity

The energy at any point of height in a rectangular channel is

$$e = \frac{\bar{U}^2}{2\,g} + \frac{P}{\rho} + z \tag{5.1}$$

why? explain

and, since $\frac{P}{\rho} + z = y$ for any point in the cross section (free surface),

$$e = y + \frac{\bar{U}^2}{2g} \tag{5.2}$$

where:

e	specific energy per unit
y	height of the liquid in the channel
g	acceleration of gravity
$\bar{U}$	average velocity of the liquid

If the velocity of the liquid is increased, the height, y, has to change to keep the same flow rate $Q = q\,b = b\,y\,\bar{U}$. For a specific flow rate and cross section, there are many combinations of velocity and height. Plotting these points on a diagram, with the y–coordinate as the height and the x–coordinate as the specific energy, e, creates a parabola on a graph. This line is known as the "specific energy curve". Several conclusions can be drawn from Figure **??**. First, there is a minimum energy at a specific height known as the "critical height". Second, the energy increases with a decrease in the height when the liquid height is below the critical height. In this case, the main contribution to the energy is due to the increase in the velocity. This flow is known as the "supercritical flow". Third, when the height is above the critical height, the energy increases again. This flow is known as the "subcritical flow", and the energy increase is due to the hydrostatic pressure component.

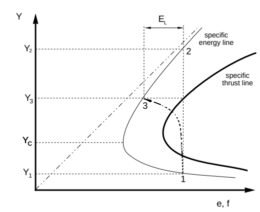


Fig. -5.2. Specific Energy and momentum Curves.

The minimum point of energy curve happens to be at

$$\bar{U} = \sqrt{g\,y_c} \tag{5.3}$$

where the critical height is defined by

$$y_c = \sqrt[3]{\frac{q^2}{g}} \tag{5.4}$$

Thrust is defined as

$$f = \frac{y^2}{2} + y\frac{\bar{U}^2}{g} \tag{5.5}$$

The minimum thrust also happens to be at the same point $\bar{U} = \sqrt{g\,y}$. Therefore, one can define the dimensionless number as:

$$Fr = \frac{g\,y_h}{\bar{U}^2} \tag{5.6}$$

Dividing the velocity by $\sqrt{gy}$ provides one with the ability to check if the flow is above or below critical velocity. This quantity is very important, and its significance can be studied from many books on fluid mechanics. The gravity effects are "measured" by the Froude number which is defined by equation (5.6).

5.2 Typical diagrams

5.3 Hydraulic Jump

The flow can change only from a supercritical flow to a subcritical flow, in which the height increases and the velocity decreases. There is no possibility for the flow to go in the reverse direction because of the Second Law of Thermodynamics (the explanation of which is out of the scope of this discussion). If there is no energy loss, the flow moves from point 1 to point 2 in Figure **??**. In actuality, energy loss occurs in any situation, but sometimes it can be neglected in the calculations. In cases where the flow changes rapidly (such as with the hydraulic jump), the energy loss **must** be taken into account. In these cases, the flow moves from point 1 to point 3 and has energy loss (E_L). In many cases the change in the thrust is negligible, such as the case of the hydraulic jump, and the flow moves from point 1 to point 3 as shown in Figure **??**.

In 1981, Garber "found" the hydraulic jump in the shot sleeve which he called a "wave". Garber built a model to describe this wave, utilizing mass conservation and Bernoulli's equation (energy conservation). This model gives a set of equations relating plunger velocity and wave velocity to other geometrical properties of the shot sleeve. Over 150 years earlier, Bélanger [**?**] demonstrated that the energy is dissipated, and that energy conservation models cannot be used to solve hydraulic jump. He demonstrated that the dissipation increases with the increase of the liquid velocity before the jump. This conclusion is true for any kind of geometry.

A literature review demonstrates that the hydraulic jump in a circular cross–section (like in a shot sleeve) appears in other cases, for example a flow in a storm sewer systems. An analytical solution that describes the solution is Bar–Meir's formula and is shown in Figure 8.4.

The energy loss concept manifests itself in several designs, such as in the energy–dissipating devices, in which hydraulic jumps are introduced in order to dissipate energy. The energy–dissipating devices are so common that numerous research works have been performed on them in the last 200 hundreds years. An excellent report by the U.S. Bureau of Reclamation [7] shows the percentage of energy loss. However, Garber, and later other researchers from Ohio State University [8], failed to know/understand/use this information.

CHAPTER 6

Runner Design

Under construction please ignore the rest of the chapter.

6.1 Introduction

In this chapter the design and the different relationship between runner segments are studied herein. The first step in runner design is to divide the mold into several logical sections. The volume of every section has to be calculated. Then the design has to ensure that the gate velocity and the filling time of every section to be as recommend by experimental results. At this stage there is no known reliable theory/model known to the author to predict these values. The values are based heavily on semi-reliable experiments. The Backward Design is discussed. The reader with knowledge in electrical engineering (electrical circles) will notice in some similarities. However, hydraulic circuits are more complex. Part of the expressions are simplified to have analytical expressions. Yet, in actuality all the terms should be taken into considerations and commercial software such DiePerfectshould be used.

6.1.1 Backward Design

Suppose that we have *n* sections with *n* gates. We know that volume to be delivered at gate *i* and is denoted by vi.The gate velocity has to be in a known range. The filling time has to be in a known function and we recommend to use Eckert/Bar–Meir's formula. For this discussion it is assumed that the filling has to be in known range and the flow rate can be calculated by

$$Q_i = \frac{V_i}{t_i} \tag{6.1}$$

Thus, gate area for the section

$$A_i = \frac{Q_i}{U_{gate}} \tag{6.2}$$

Armed with this knowledge, one can start design the runner system.

6.1.2 Connecting runner segments

Design of connected runner segments have insure that the flow rate at each segment has to be designed one. In Figure **??**a branches *i* and *j* are connected to branch κ at point K. The pressure drop (difference) on branches *i* and *j* has to be the same since the pressure in the mold cavity is the same for both segments. The sum of the flow rates for both branch has to be equal to flow rate in branch κ

$$Q_\kappa = Q_i + Q_j \Longrightarrow Q_j = Q_\kappa - Q_i \tag{6.3}$$

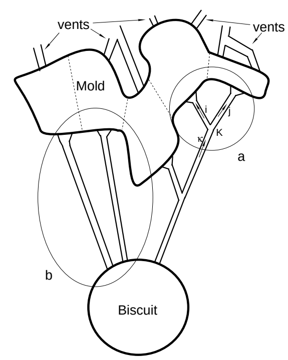


Fig. -6.1. A geometry of runner connection.

The flow rate in every branch is related to the pressure difference by

$$Q_i = \frac{\Delta P}{R_i} \tag{6.4}$$

Where the subscript *i* in this case also means any branch e.g. *i*, *j* and so on. For example, one can write for branch *j*

$$O_j = \frac{DeltaP}{R_j} \tag{6.5}$$

Utilizing the mass conservation for point K in which $Q_{kappa} = O_i + O_j$ and the fact that the pressure difference, $DeltaP$, is the same thus we can write

$$Q_k = \frac{DeltaP}{R_i} + \frac{DeltaP}{R_j} = DeltaP \frac{R_i R_j}{R_i + R_j} \tag{6.6}$$

where we can define equivalent resistance by

$$\bar{R} = \frac{R_i R_j}{R_i + R_j} \tag{6.7}$$

Lets further manipulate the equations to get some more important relationships. Using equation (**??**) and equation (**??**)

$$DeltaP_i = DeltaP_j \Longrightarrow O_i R_i = O_j R_j \tag{6.8}$$

The flow rate in a branch *j* can be related to flow rate in branch *i* and corresponding resistances

$$O_j = \frac{R_i}{R_j} O_i \tag{6.9}$$

Using equation (**??**) and equation (**??**) one can obtain

$$\frac{O_i}{Q_k} = \frac{R_j}{R_j + Ri} \tag{6.10}$$

Solving for the resistance ratio since the flow rate is known

$$\frac{R_i}{R_j} = \frac{O_i}{Q_k} - 1 \tag{6.11}$$

6.1.3 Resistance

What does the resistance include? How to achieve resistance ratio in the previous equation (**??**) will be discussed herein further. The total resistance reads

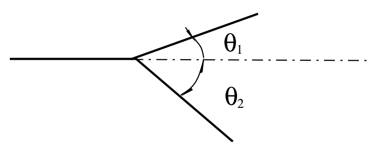


Fig. -6.2. y connection.

$$R = R_{ii} + R_\theta + R_{geometry} + R_{contraction} + R_{ki} + R_{exit} \tag{6.12}$$

The contraction resistance, $R_{contraction}$, is the due the contraction of the gate. The exit resistance, R_{exit}, is due to residence of the liquid metal in mold cavity. Or in other words, the exit resistance is due the lost of energy of immersed jet. The angle resistance, R_θ is due to the change of direction. The R_{ki} is the resistance due to flow in the branch κ on branch i. The geometry resistance $R_{geometry}$, is due to who rounded the connection.

$$\frac{DeltaP}{\rho} = f \frac{L}{H_D} \frac{U^2}{2} \tag{6.13}$$

since $U_i = \frac{O_i}{A}$

$$\frac{DeltaP}{\rho} = f \frac{L}{H_D} \frac{{O_i}^2}{2A^2} \tag{6.14}$$

$$\frac{DeltaP}{\rho} = (C) f \frac{L}{{H_D}^3} \frac{{O_i}^2}{2} \tag{6.15}$$

Lets assume further that $L_i = L_j$, $\frac{O_i}{O_j} = known$

$$f_i = f_j = f \tag{6.16}$$

$$(C) f \frac{L}{{H_D}^3{}_i} \frac{{O_i}^2}{2} = (C) f \frac{L}{{H_D}^3{}_j} \frac{{O_j}^2}{2} \tag{6.17}$$

$$\frac{H_D i}{H_{Dj}} = \left(\frac{O_i}{O_j}\right)^{\frac{2}{3}} \tag{6.18}$$

Comparison between scrap between (multi-lines) two lines to one line
first find the diameter equivalent to two lines

$$DeltaP = (C) f \frac{L}{2} \frac{{Q_k}^2}{{H_D}^3_k} = (C) f \frac{L}{2} \frac{(O_i + O_j)^2}{{H_D}^3_k} \tag{6.19}$$

$$O_i{}^2 = \frac{DeltaP}{f} \frac{2}{L} H_D i^3 \tag{6.20}$$

subtitling in to

$$\overline{H_{Dk}} = \sqrt[3]{({H_D}_i^3 + {H_D}_j^3} \tag{6.21}$$

Now we know the relationship between the hydraulic radius. Let see what is the scrap difference between them.

put drawing of the trapezoid
let scrap denoted by η
converting the equation

$$H_D i = \sqrt[3/2]{\frac{\eta_i}{constL}} \tag{6.22}$$

the ratio of the scrap is

$$\frac{\eta_i + \eta_j}{\eta_k} = \frac{(H_D i + 2 + H_D j^2)}{H_D k^2} \tag{6.23}$$

and now lets write $H_D k$ in term of the two other

$$\frac{({H_D}_i^2 + {H_D}_j^2)}{\left({H_D}_i^3 + {H_D}_j^3\right)^{2/3}} \tag{6.24}$$

CHAPTER 7

PQ² Diagram Calculations

In conclusion, it's just a plain sloppy piece of work.

Referee II, see In the appendix B

7.1 Introduction

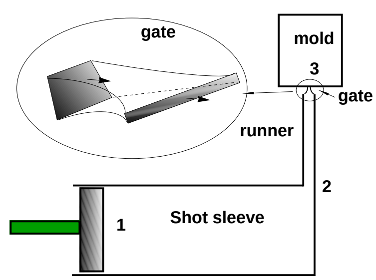


Fig. -7.1. Schematic of typical die casting machine.

The pQ^2 diagram is the most common calculation, if any at all, are used by most die casting engineers. The importance of this diagram can be demonstrated by the fact that tens of millions of dollars have been invested by NADCA, NSF, and other major institutes here and abroad in the pQ^2 diagram research. The pQ^2 diagram is one of the manifestations of supply and demand theory which was developed by Alfred Marshall (1842–1924) in the turn of the century. It was first introduced to the die casting industry in the late'70s [12]. In this diagram, an engineer insures that die casting machine ability can fulfill the die mold design requirements; the liquid metal is injected at the right velocity range and the filling time is small enough to prevent premature freezing. One can, with the help of the pQ^2 diagram, and by utilizing experimental values for desired filling time and gate velocities improve the quality of the casting.

In the die casting process (see Figure 7.1), a liquid metal is poured into the shot sleeve where it is propelled by the plunger through the runner and the gate into the mold. The gate thickness is very narrow compared with the averaged mold thickness and the runner thickness to insure that breakage point of the scrap occurs at that gate location. A solution of increasing the discharge coefficient, C_D, (larger conduits) results

perhaps put this section in general discussion in a larger scrap. A careful design of the runner and the gate is required.

First, the "common" pQ^2 diagram[1] is introduced. The errors of this model are analyzed. Later, the reformed model is described. Effects of different variables is studied and questions for students are given in the end of the chapter.

7.2 The "common" pQ^2 diagram

The injection phase is (normally) separated into three main stages which are: slow part, fast part and the intensification (see Figure 7.2). In the slow part the plunger moves in the critical velocity to prevent wave formation and therefore expels maximum air/gas before the liquid metal enters the cavity. In the fast part the cavity supposed to be filled in such way to prevent premature freezing and to obtain the right filling pattern. The intensification part is to fill the cavity with additional material to compensate for the shrinkage porosity during the solidification process. The pQ^2 diagram deals with the second part of the filling phase.

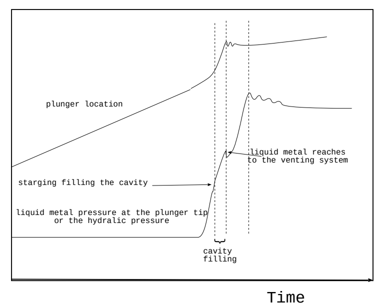


Fig. -7.2. A typical trace on a cold chamber machine

In the pQ^2 diagram, the solution is determined by finding the intersecting point of the runner/mold characteristic line with the pump (die casting machine) characteristic line. The intersecting point sometime refereed to as the operational point. The machine characteristic line is assumed to be understood to some degree and it requires finding experimentally two coefficients. The runner/mold characteristic line requires knowledge on the efficiency/discharge coefficient, C_D, thus it is an essential parameter in the calculations. Until now, C_D has been evaluated either experimentally, to be assigned to specific runner, or by the liquid metal properties ($C_D \propto \rho$) [9] which is de facto the method used today and refereed herein as the "common" pQ^2 diagram[2]. Furthermore, C_D is assumed constant regardless to any change in any of the machine/operation parameters during the calculation. The experimental approach is arduous and expensive, requiring the building of the actual mold for each attempt with average cost of $5,000–$10,000 and is rarely used in the industry[3]. A short discussion about this issue is presented in the Appendix B comments to referee 2.

Herein the "common" model (constant C_D) is constructed. The assumptions made in the construction of the model as following

---

[1]as this model is described in NADCA's books

[2]Another method has been suggested in the literature in which the C_D is evaluated based on the volume to be filled [10]. The author does not know of anyone who use this method and therefore is not discussed in this book. Nevertheless, this method is as "good" as the "common" method.

[3]if you now of anyone who use this technique please tell me about it.

1. C_D assumed to be constant and depends only the metal. For example, NADCA recommend different values for aluminum, zinc and magnesium alloys.

2. Many terms in Bernoulli's equation can be neglected.

3. The liquid metal is reached to gate.

4. No air/gas is present in the liquid metal.

5. No solidification occurs during the filling.

6. The main resistance to the metal flow is in the runner.

7. A linear relationship between the pressure, P_1 and flow rate (squared), Q^2.

According to the last assumption, the liquid metal pressure at the plunger tip, P_1, can be written as

$$P_1 = P_{max} \left[1 - \left(\frac{Q}{Q_{max}} \right)^2 \right] \tag{7.1}$$

Where:

P_1	the pressure at the plunger tip
Q	the flow rate
$\P_{max}$	maximum pressure which can be attained by the die casting machine in the shot sleeve
Q_{max}	maximum flow rate which can be attained in the shot sleeve

The P_{max} and Q_{max} values to be determined for every set of the die casting machine and the shot sleeve. The P_{max} value can be calculated using a static force balance. The determination of Q_{max} value is done by measuring the velocity of the plunger when the shot sleeve is empty. The maximum velocity combined with the shot sleeve cross–sectional area yield the maximum flow rate,

$$Q_i = A \times U_i \tag{7.2}$$

where i represent any possible subscription e.g. $i = max$

Thus, the first line can be drawn on pQ^2 diagram as it shown by the line denoted as *1* in Figure **??**. The line starts from a higher pressure (P_{max}) to a maximum flow rate (squared). A new combination of the same die casting machine and a different plunger diameter creates a different line. A smaller plunger diameter has a larger maximum pressure (P_{max}) and different maximum flow rate as shown by the line denoted as *2*.

The maximum flow rate is a function of the maximum plunger velocity and the plunger diameter (area). The plunger area is a obvious function of the plunger diameter, $A = \pi D^2/4$. However, the maximum plunger velocity is a far–more complex function. The force that can be extracted from a die casting machine is essentially the same for different plunger diameters. The change in the resistance as results of changing the plunger (diameter) depends on the conditions of the plunger. The "dry" friction will be same what different due to change plunger weight, even if the plunger conditions where the same. Yet, some researchers claim that plunger velocity is almost invariant in regard to the plunger diameter[4]. Nevertheless, this piece of information has no bearing on the derivation in this model or reformed one, since we do not use it.

Example 7.1:
*Prove that the maximum flow rate, Q_{max} is reduced and that $Q_{max} \propto 1/{D_P}^2$ (see Figure **??**). if U_{max} is a constant*

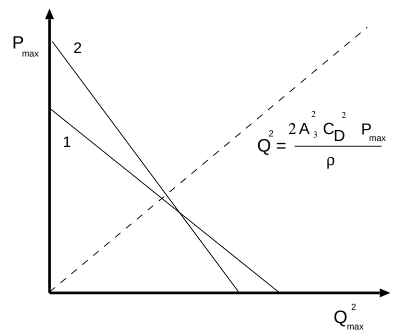


Fig a. *The "common" pQ² version.*

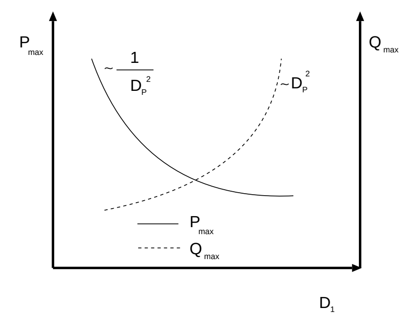


Fig b. *P_{max} and Q_{max} ass a function of the plunger diameter according to "common" model.*

Fig. -7.3. The left graph depicts the "common" pQ^2 version. The right graph depicts P_{max} and Q_{max} as a function of the plunger diameter according to "common" model.

A simplified force balance on the rode yields (see more details in section 7.11 page 97)

$$P_{max} = P_B \left(\frac{D_B}{D_1}\right)^2 = \frac{P_B}{{D_1}^2} {D_B}^2 \tag{7.3}$$

where subscript B denotes the actuator.

What is the pressure at the plunger tip when the pressure at the actuator is 10 [bars] with diameter of 0.1[m] and with a plunger diameter, D_1, of 0.05[m]? Substituting the data into equation (7.3) yields

$$P_1 = 10 \times \left(\frac{0.1}{0.05}\right)^2 = 4.0[MPa]$$

[4]More research is need on this aspect.
++ read the comment made by referee II to the paper on pQ² on page 194.

In the "common" pQ^2 diagram C_D is defined as

$$C_D = \sqrt{\frac{1}{1+K_F}} = constant \tag{7.4}$$

Note, therefore K_F is also defined as a constant for every metal[5]. Utilizing Bernoulli's equation[6].

$$U_3 = C_D \sqrt{\frac{2P_1}{\rho}} \tag{7.5}$$

The flow rate at the gate can be expressed as

$$Q_3 = A_3 C_D \sqrt{\frac{2P_1}{\rho}} \tag{7.6}$$

The flow rate in different locations is a function of the temperature. However, Eckert [7] demonstrated that the heat transfer is insignificant in the duration of the filling of the cavity, and therefore the temperature of the liquid metal can be assumed almost constant during the filling period (which in most cases is much less 100 milliseconds). As such, the solidification is insignificant (the liquid metal density changes less than 0.1% in the runner); therefore, the volumetric flow rate can be assumed constant:

to make question about mass balance

$$Q_1 = Q_2 = Q_3 = Q \tag{7.7}$$

Hence, we have two equations (7.1) and (7.6) with two unknowns (Q and P_1) for which the solution is

$$P_1 = \frac{P_{max}}{1 - \frac{2{C_D}^2 P_{max} {A_3}^2}{\rho {Q_{max}}^2}} \tag{7.8}$$

insert a discussion in regards to the trends

insert the calculation with respect to $\frac{dU_3}{odA_1}$ and $\frac{dP_1}{dA_1}$

7.3 The validity of the "common" diagram

In the construction of the "common" model, two main assumptions were made: one C_D is a constant which depends only on the liquid metal material, and two) many terms in the energy equation (Bernoulli's equation) can be neglected. Unfortunately, the examination of the validity of these assumptions was missing in all the previous studies. Here, the question when the "common" model valid or perhaps whether the "common' model valid at all is examined. Some argue that even if the model is wrong and do not stand on sound scientific principles, it still has a value if it produces reasonable trends. Therefore, this model should produce reasonable results and trends when varying any parameter in order to have any value. Part of the examination is done by varying parameters and checking to see what happen to trends.

[5] The author would like to learn who came–out with this "clever" idea.
[6] for more details see section 7.4 page 87.
[7] read more about it in Chapter 3.

7.3.1 Is the "Common" Model Valid?

Is the mass balance really satisfied in the "common" model? Lets examine this point. Equation (7.7) states that the mass (volume, under constant density) balance is exist.

$$A_1U_1 = A_3U_3 \tag{7.9}$$

So, what is the condition on C_D to satisfy this condition? Can C_D be a constant as stated in assumption 1? To study this point let derive an expression for C_D. Utilizing equation (7.5) yields

$$A_1\,U_1 = A_3C_D\sqrt{\frac{2\,P_1}{\rho}} \tag{7.10}$$

From the machine characteristic, equation (7.1), it can be shown that

$$U_1 = U_{max}\sqrt{\frac{P_{max}-P_1}{P_{max}}} \tag{7.11}$$

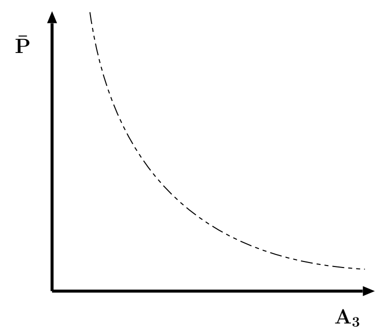


Fig. -7.4. $\bar{P}$ as A_3 to be relocated

Example 7.2:
Derive equation 7.11. Start with machine characteristic equation (7.1)

Substituting equation (7.11) into equation (7.10) yield,

$$A_1U_{max}\sqrt{\frac{P_{max}-P_1}{P_{max}}} = A_3C_D\sqrt{\frac{P_1}{\rho}} \tag{7.12}$$

It can be shown that equation (7.12) can be transformed into

$$C_D = \frac{A_1}{A_3}\frac{U_{max}\sqrt{\rho}}{\sqrt{\frac{2\,P_1\,P_{max}}{P_{max}-P_1}}} \tag{7.13}$$

Example 7.3:
Find the relationship between C_D and Ozer number that satisfy equation (7.13)

According to the "common" model U_{max}, and P_{max} are independent of the gate area, A_3. The term $A_3\sqrt{\frac{P_1}{P_{max}-P_1}}$ is not a constant and is a function of A_3 (possibility other parameters).

Example 7.4:

find the relationship between $\left[A_3\sqrt{\frac{P_1}{P_{max}-P_1}}\ \right]$ *and* A_3

Solution

under construction

End Solution

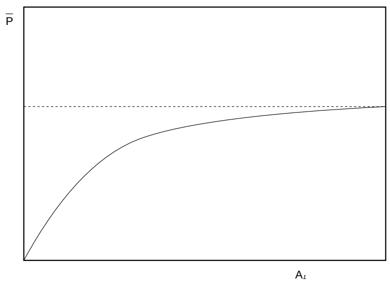


Fig a. $\bar{P}$ *as* A_1 *to be relocated*

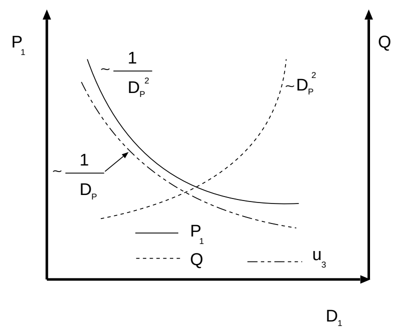


Fig b. P_1, Q, *and* U_3 *as a function of plunger diameter ,* A_1.

Fig. -7.5. Pressure at the plunger tip, P_1*, the flow rate,* Q*, and the gate velocity,* U_3 *as a function of plunger diameter ,* A_1.

Example 7.5:

A_3 *what other parameters that* C_D *depend on which do not provide the possibility* $C_D = constant$*?*

To maintain the mass balance C_D must be a function at least of the gate area, A_3. Since the "common" pQ^2 diagram assumes that C_D is a constant it diametrically opposite the mass conservation principle. Moreover, in the "common" model, a major assumption is that the value of C_D depends on the metal, therefore, the mass balance is probably never achieved in many cases. This violation demonstrates, once for all, that the "common" pQ^2 diagram is erroneous.

Solution

under construction.

End Solution

Use the information from example **??** and check what happened to the flow rate at two location (1) gate 2) plunger tip) when discharge coefficient is varied $C_D = 0.4-0.9$

7.3.2 Are the Trends Reasonable?

Now second part, are the trends predicted by the "common" model are presumable (correct)? To examine that, we vary the plunger diameter, (A_1 or D_1) and the gate area, A_3 to see if any violation of the physics laws occurs as results. The comparison between the real trends and the "common" trends is discussed in the following section.

Plunger area/diameter variation

First, the effect of plunger diameter size variation is examined. In section 7.2 it was shown that $P_{max} \propto 1/{D_1}^2$. Equation (7.8) demonstrates that P_1 increases with an increase of P_{max}. It also demonstrates that the value of $\bar{P}$ never can exceed

$$\left[\frac{P_1}{P_{max}}\right]_{max} = \frac{\rho}{2}\left(\frac{Q_{max}}{C_D\, A_3}\right)^2 \tag{7.14}$$

The value P_{max} can attained is an infinite value (according to the "common" model) therefore P_1 is infinite as well. The gate velocity, U_3, increases as the plunger diameter decreases as shown in Figure **??**. Armed with this knowledge now, several cases can be examined if the trends are realistic.

Gate area variation

Energy conservation (power supply machine characteristic) Let's assume that mass conservation is fulfilled, and, hence the plunger velocity can approach infinity, $U_1 \rightarrow \infty$ when $D_1 \rightarrow 0$ (under constant Q_{max}). The hydraulic piston also has to move with the same velocity, U_1. Yet, according to the machine characteristic the driving pressure, approaches zero $(P_{B1} - P_{B2}) \rightarrow 0$. Therefore, the energy supply to the system is approaching zero. Yet, energy obtained from the system is infinite since jet is inject in infinite velocity and finite flow rate. This cannot exist in our world or perhaps one can proof the opposite.

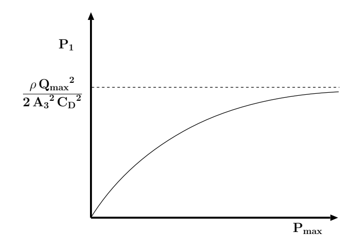


Fig. -7.6. P_1 as a function of P_{max}.

Energy conservation (power supply) Assuming that the mass balance requirement is obtained, the pressure at plunger tip, P_1 and gate velocity, U_3, increase (and can reach infinity,(when $P_1 \rightarrow \infty$ then $U_3 \rightarrow \infty$) when the plunger diameter is reduced. Therefore, the energy supply to the system has to be infinity (assuming a constant energy dissipation, actually the dissipation increases with plunger diameter in most ranges) However, the energy supply to the system (c.v. only the liquid metal) system would be $P_{B1}A_{B1}U_1$ (finite amount) and the energy the system provide plus would be infinity (infinite gate velocity) plus dissipation.

to make a question in regards to dissipation and velocity

Energy conservation (dissipation problem) A different way to look at this situation is check what happen to physical quantities. For example, the resistance to the liquid metal flow increases when the gate velocity velocity is increased. As smaller the plunger diameter the larger the gate velocity and the larger the resistance. However, the energy supply to the system has a maximum ability. Hence, this trend from this respect is unrealistic.

Mass conservation (strike) According to the "common" model, the gate velocity decreases when the plunger diameter increases. Conversely, the gate velocity increases when the plunger diameter decreases[8]. According to equation (7.2) the liquid metal flow rate at the gate increases as well. However, according to the "common" pQ^2 diagram, the plunger can move only in a finite velocity lets say in the extreme case U_{max} [9]. Therefore, the flow rate at the plunger tip decreases. Clearly, these diametrically opposing trends cannot coexist. Either the "common" pQ^2 diagram wrong or the mass balance concept is wrong, take your pick.

Mass conservation (hydraulic pump): The mass balance also has to exist in hydraulic pump (obviously). If the plunger velocity have to be infinite to maintain mass balance in the metal side, the mass flow rate at the hydraulic side of the rode also have to be infinite. However, the pump has maximum capacity for flow rate. Hence, mass balance can be obtained.

to put table with different trends as a function of A_3 and may be with a figure.

7.3.3 Variations of the Gate area, A_3

under construction

7.4 The reformed pQ^2 diagram

The method based on the liquid metal properties is with disagreement with commonly agreed on in fluid mechanics [27, pp. 235-299]. It is commonly agreed that C_D is a function of Reynolds number and the geometry of the runner design. The author suggested adopting an approach where the C_D is calculated by utilizing data of flow resistance of various parts (segments) of the runner. The available data in the literature demonstrates that a typical value of C_D can change as much as 100% or more just by changing the gate area (like valve opening). Thus, the assumption of a constant C_D, which is used in "common" pQ^2 calculations[10], is not valid. Here a systematic derivation of the pQ^2 diagram is given. The approach adapted in this book is that everything (if possible) should be presented in dimensionless form.

[8] check again Figure **??**

[9] this is the velocity attained when the shot sleeve is empty

[10] or as it is suggested by the referee II

7.4.1 The reform model

Equation (7.1) can be transformed into dimensionless from as

$$\bar{Q} = \sqrt{1-\bar{P}} \tag{7.15}$$

Where:

$\bar{P}$	reduced pressure, P_1/P_{max}
$\bar{Q}$	reduced flow rate, Q_1/Q_{max}

Eckert also demonstrated that the gravity effects are negligible[11]. Assuming steady state[12] and utilizing Bernoulli's equation between point (1) on plunger tip and point (3) at the gate area (see Figure 7.1) yields

$$\frac{P_1}{\rho} + \frac{{U_1}^2}{2} = \frac{P_3}{\rho} + \frac{{U_3}^2}{2} + h_{1,3} \tag{7.16}$$

where:

U	velocity of the liquid metal
ρ	the liquid metal density
$h_{1,3}$	energy loss between plunger tip and gate exit
subscript	
1	plunger tip
2	entrance to runner system
3	gate

It has been shown that the pressure in the cavity can be assumed to be about atmospheric (for air venting or vacuum venting) providing vents are properly designed Bar–Meir at el [13]. This assumption is not valid when the vents are poorly designed. When they are poorly designed, the ratio of the vent area to critical vent area determines the build up pressure, P_3, which can be calculated as it is done in Bar–Meir et al However, this is not a desirable situation since a considerable gas/air porosity is created and should be avoided. It also has been shown that the chemical reactions do not play a significant role during the filling of the cavity and can be neglected [5].

The resistance in the mold to liquid metal flow depends on the geometry of the part to be produced. If this resistance is significant, it has to be taken into account calculating the total resistance in the runner. In many geometries, the liquid metal path in the mold is short, then the resistance is insignificant compared to the resistance in the runner and can be ignored. Hence, the pressure at the gate, P_3, can be neglected. Thus, equation (7.16) is reduced to

$$\frac{P_1}{\rho} + \frac{{U_1}^2}{2} = \frac{{U_3}^2}{2} + h_{1,3} \tag{7.17}$$

[11] see for more details chapter 3

[12] read in the section 7.4.4 on the transition period of the pQ^2

[13] Read a more detailed discussion in Chapter 9

The energy loss, $h_{1,3}$, can be expressed in terms of the gate velocity as

$$h_{1,3} = K_F \frac{U_3^2}{2} \tag{7.18}$$

where K_F is the resistance coefficient, representing a specific runner design and specific gate area.

Combining equations (7.7), (7.17) and (7.18) and rearranging yields

$$U_3 = C_D \sqrt{\frac{2P_1}{\rho}} \tag{7.19}$$

where

$$C_D = f(A_3, A_1) = \sqrt{\frac{1}{1 - \left(\frac{A_3}{A_1}\right)^2 + K_F}} \tag{7.20}$$

Converting equation (7.19) into a dimensionless form yields

$$\bar{Q} = \sqrt{2Oz\bar{P}} \tag{7.21}$$

When the Ozer Number is defined as

$$Oz = \frac{\frac{{C_D}^2 P_{max}}{\rho}}{\left(\frac{Q_{max}}{A_3}\right)^2} = \left(\frac{A_3}{Q_{max}}\right)^2 {C_D}^2 \frac{P_{max}}{\rho} \tag{7.22}$$

The significance of the Oz number is that this is the ratio of the "effective" maximum energy of the hydrostatic pressure to the maximum kinetic energy. Note that the Ozer number is not a parameter that can be calculated a priori since the C_D is varying with the operation point.

[14] For practical reasons the gate area, A_3 cannot be extremely large. On the other hand, the gate area can be relatively small $A_3 \sim 0$ in this case Ozer number $A_3 {A_3}^n$ where is a number larger then 2 ($n > 2$).

Solving equations (7.21) with (7.15) for $\bar{P}$, and taking only the possible physical solution, yields

$$\bar{P} = \frac{1}{1 + 2\,Oz} \tag{7.23}$$

which is the dimensionless form of equation (7.8).

[14]It should be margin-note and so please ignore this footnote.
how Ozer number behaves as a function of the gate area?

$$Oz = \frac{P_{max}}{\rho {Q_{max}}^2} \frac{A_3}{1 - \left(\frac{A_3}{A_1}\right)^2 + K_F}$$

7.4.2 Examining the solution

This solution provide a powerful tool to examine various parameters and their effects on the design. The important factors that every engineer has to find from these calculations are: gate area, plunger diameters, the machine size, and machine performance etc[15]. These issues are explored further in the following sections.

The gate area effects

Gate area affects the reduced pressure, $\bar{P}$, in two ways: via the Ozer number which include two terms: one, (A_3/Q_{max}) and, two, discharge coefficient C_D. The discharge coefficient, C_D is also affected by the gate area affects through two different terms in the definition (equation 7.20), one, $(A_3/A_1)^2$ and two by K_F.

perhaps to put discussion pending on the readers response.

Q_{max} effect is almost invariant with respect to the gate area up to small gate area sizes[16]. Hence this part is somewhat clear and no discussion is need.

$(A_3/A_1)^2$ effects Lets look at the definition of C_D equation (7.20). For illustration purposes let assume that K_F is not a function of gate area, $K_F(A_3) = constant$. A small perturbation of the gate area results in Taylor series,

$$\begin{aligned}
\Delta C_D &= C_D(A_3 + \Delta A_3) - C_D(A_3) \\
&= \frac{1}{\sqrt{1 - \frac{{A_3}^2}{{A_1}^2} + K_F}} + \frac{A_3\,\Delta A_3}{{A_1}^2\left(1 - \frac{{A_3}^2}{{A_1}^2} + K_F\right)^{\frac{3}{2}}} + \\
&\quad \frac{\left(\frac{3\,{A_3}^2}{{A_1}^4\left(1-\frac{{A_3}^2}{{A_1}^2}+K_F\right)^2} + \frac{1}{{A_1}^2\left(1-\frac{{A_3}^2}{{A_1}^2}+K_F\right)}\right){\Delta A_3}^2}{2\sqrt{1 - \frac{{A_3}^2}{{A_1}^2} + K_F}} + \mathbf{O}(\Delta A_3)^3
\end{aligned} \tag{7.24}$$

In this case equation (7.8) still hold but C_D has to be reevaluated. repeat the example **??** with $K_F = 3.3$ First calculate the discharge coefficient, C_D for various gate area starting from 2.4 10^{-6} $[m^2]$ to 3 10^{-4} $[m^2]$ using equation 7.20.

This example demonstrate the very limited importance of the inclusion of the term $(A_3/A_1)^2$ into the calculations.

K_F effects The change in the gate area increases the resistance to the flow via several contributing factors which include: the change in the flow cross section, change in the direction of the flow, frictional loss due to flow through the gate length, and the loss due to the abrupt expansion after the gate. The loss due to the abrupt expansion is a major contributor and its value changes during the filling process. The liquid metal

[15]The machine size also limited by a second parameter known as the clamping forces to be discussed in Chapter 11

[16]This is reasonable speculation about this point. More study is well come

enters the mold cavity in the initial stage as a "free jet" and sometime later it turns into an immersed jet which happens in many geometries within 5%-20% of the filling. The change in the flow pattern is believed to be gradual and is a function of the mold geometry. A geometry with many changes in the direction of the flow and/or a narrow mold (relatively thin walls) will have the change to immersed jet earlier. Many sources provide information on K_F for various parts of the designs of the runner and gate. Utilizing this information produces the gate velocity as a function of the given geometry. To study further this point consider a case where K_F is a simple function of the gate area.

When A_3 is very large then the effect on K_F are relatively small. Conversely, when $A_3 \to 0$ the resistance, $K_F \to \infty$. The simplest function, shown in Figure 7.7, that represent such behavior is

$$K_F = C_1 + \frac{C_2}{A_3} \tag{7.25}$$

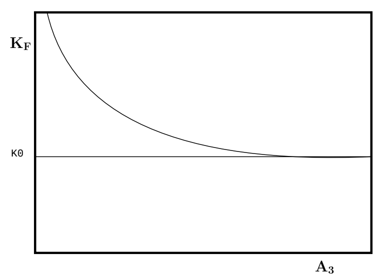


Fig. -7.7. K_F as a function of gate area, A_3

C_1 and C_2 are constants and can be calculated (approximated) for a specific geometry. The value of C_1 determine the value of the resistance where A_3 effect is minimum and C_2 determine the range (point) where A_3 plays a significant effect. In practical, it is found that C_2 is in the range where gate area are desired and therefore program such as DiePerfect™ are important to calculated the actual resistance.

Example 7.1:

Under construction
create a question with respect to the function 7.25

Solution

Under construction

The combined effects Consequently, a very small area ratio results in a very large resistance, and when $\frac{A_3}{A_1} \to 0$ therefore the resistance $\to \infty$ resulting in a zero gate velocity (like a closed valve). Conversely, for a large area ratio, the resistance is insensitive to variations of the gate area and the velocity is reduced with increase the gate area. Therefore a maximum gate velocity must exist, and can be found by

$$\frac{dU_3}{dA_3} = 0 \tag{7.26}$$

which can be solved numerically. The solution of equation (7.26) requires full information on the die casting machine.

A general complicated runner design configuration can be converted into a straight conduit with trapezoidal cross–section, provided that it was proportionally designed leave it for now, better presentation needed

to create equal gate velocity for different gate locations[17]. The trapezoidal shape is commonly used because of the simplicity, thermal, and for cost reasons. To illustrate only the effects of the gate area change two examples are presented: one, a constant pressure is applied to the runner, two, a constant power is applied to the runner. The resistance to the flow in the shot sleeve is small compared to resistance in the runner, hence, the resistance in the shot sleeve can be neglected. The die casting machine performance characteristics are isolated, and the gate area effects on the the gate velocity can be examined. Typical dimensions of the design are presented in Figure **??**. The short conduit of 0.25[m] represents an excellent runner design and the longest conduit of 1.50[m] represents a very poor design. The calculations were carried for aluminum alloy with a density of 2385[kg/m^3] and a kinematic viscosity of 0.544×10^{-6}[m^2/sec] and runner surface roughness of 0.01 [mm]. For the constant pressure case the liquid metal pressure at the runner entrance is assumed to be 1.2[MPa] and for the constant power case the power loss is [1Kw]. filling time that

$$t_{max} \geq t = \frac{V}{Q_{max}\sqrt{2Oz^{*}\bar{P}^{*}}} \tag{7.27}$$

The gate velocity is exhibited as a function of the ratio of the gate area to the conduit area as shown in Figure **??** for a constant pressure and in Figure **??** for a constant power.

General conclusions from example 7.7

For the constant pressure case the "common"[18] assumption yields a constant velocity even for a zero gate area.

The solid line in Figure **??** represents the gate velocity calculated based on the common assumption of constant C_D while the other lines are based on calculations which take into account the runner geometry and the Re number. The results for constant C_D represent "averaged" of the other results. The calculations of the velocity based on a constant C_D value are unrealistic. It overestimates the velocity for large gate area and underestimates for the area ratio below $\sim$ 80% for the short runner and 35% for the long runner. Figure **??** exhibits that there is a clear maximum gate velocity which depends on the runner design (here represented by the conduit length). The maximum indicates that the preferred situation is to be on the "right hand side branch" because of shorter filling time. The gate velocity is doubled for the excellent design compared with the gate velocity obtained from the poor design. This indicates that the runner design is more important than the specific characteristic of the die casting machine performance. Operating the die casting machine in the "right hand side" results in smaller requirements on the die casting machine because of a smaller filling time, and therefore will require a smaller die casting machine.

For the constant power case, the gate velocity as a function of the area ratio is shown in Figure **??**. The common assumption of constant C_D yields the gate velocity

[17] read about poor design effect on pQ^2 diagram

[18] As it is written in NADCA's books

$U_3 \propto A_1/A_3$ shown by the solid line. Again, the common assumption produces unrealistic results, with the gate velocity approaching infinity as the area ratio approaches zero. Obviously, the results with a constant C_D over estimates the gate velocity for large area ratios and underestimates it for small area ratios. The other lines describe the calculated gate velocity based on the runner geometry. As before, a clear maximum can also be observed. For large area ratios, the gate velocity with an excellent design is almost doubled compared to the values obtained with a poor design. However, when the area ratio approaches zero, the gate velocity is insensitive to the runner length and attains a maximum value at almost the same point.

In conclusion, this part has been shown that the use of the "common" pQ^2 diagram with the assumption of a constant C_D may lead to very serious errors. Using the pQ^2 diagram, the engineer has to take into account the effects of the variation of the gate area on the discharge coefficient, C_D, value. The two examples given inhere do not represent the characteristics of the die casting machine. However, more detailed calculations shows that the constant pressure is in control when the plunger is small compared to the other machine dimensions and when the runner system is very poorly designed. Otherwise, the combination of the pressure and power limitations results in the characteristics of the die casting machine which has to be solved.

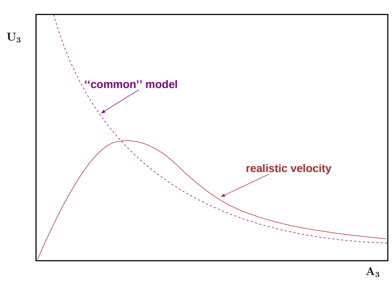


Fig a. *U_3 as a function of gate area, A_3*

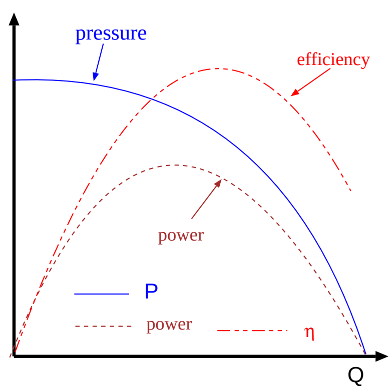


Fig b. *General characteristic of a pump.*

Fig. -7.8. Velocity, U_3 as a function of the gate area, A_3 and the general characterstic of a pump

The die casting machine characteristic effects

There are two type of operation of the die casting machine, one) the die machine is operated directly by hydraulic pump (mostly on the old machines). two) utilizing the non continuous demand for the power, the power is stored in a container and released when need (mostly on the newer machines). The container is normally a large tank contain nitrogen and hydraulic liquid[19]. The effects of the tank size and gas/liquid ratio on the pressure and flow rate can easily be derived.

Meta

The power supply from the tank with can consider almost as a constant pressure but the line to actuator is with variable resistance which is a function of the liquid velocity. The resistance can be consider, for a certain range, as a linear function of the velocity square, "${U_B}^2$". Hence, the famous a assumption of the "common" die casting machine $p \propto Q^2$.

Meta End

The characteristic of the various pumps have been studied extensively in the past [15]. The die casting machine is a pump with some improvements which are patented by different manufactures. The new configurations, such as double pushing cylinders, change somewhat the characteristics of the die casting machines. First let discuss some general characteristic of a pump (issues like impeller, speed are out of the scope of this discussion). A pump is mechanical devise that transfers and electrical power (mostly) into "hydraulic" power. A typical characteristic of a pump are described in Figure **??**.

Two similar pumps can be connect in two way series and parallel. The serious connection increase mostly the pressure as shown in Figure **??**. The series connection if "normalized" is very close to the original pump. However, the parallel connection when "normalized" show a better performance.

To study the effects of the die casting machine performances, the following functions are examined (see Figure 7.9):

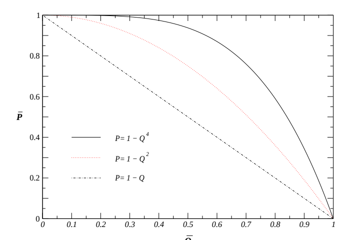


Fig. -7.9. Various die casting machine performances

$$machine : linear \bar{Q} = 1 - \bar{P} \tag{7.28}$$

$$machine : sqrt \bar{Q} = \sqrt{1 - \bar{P}} \tag{7.29}$$

$$machine : sq \bar{Q} = \sqrt[4]{1 - \bar{P}} \tag{7.30}$$

[19]This similar to operation of water system in a ship, many of the characteristics are the same. Furthermore, the same differential equations are governing the situation. The typical questions such as the necessarily container size and the ratio of gas to hydraulic liquid were part of my study in high school (probably the simplified version of the real case). If demand to this material raised, I will insert it here in the future.

The functions (**??**), (**??**) and (**??**) represent a die casting machine with a poor performance, the common performance, and a die casting machine with an excellent performance, respectively.

Combining equation (7.21) with (7.28) yields

$$1 - \bar{P} = \sqrt{2Oz\bar{P}} \tag{7.31}$$

$$\sqrt{1 - \bar{P}} = \sqrt{2Oz\bar{P}} \tag{7.32}$$

$$\sqrt[4]{1 - \bar{P}} = \sqrt{2Oz\bar{P}} \tag{7.33}$$

rearranging equation (7.31) yields

$$\bar{P}^2 - 2(1 + Oz)\bar{P} + 1 = 0 \tag{7.34}$$

$$1 - \bar{P}(1 + 2Oz) = 0 \tag{7.35}$$

$$4Oz\bar{P}^2 + \bar{P} - 1 = 0 \tag{7.36}$$

Solving equations (7.34) for $\bar{P}$, and taking only the possible physical solution, yields

$$\bar{P} = 1 + Oz - \sqrt{(2 + Oz)\,Oz} \tag{7.37}$$

$$\bar{P} = \frac{1}{1 + 2\,Oz} \tag{7.38}$$

$$\bar{P} = \frac{\sqrt{1 + 16\,Oz^2} - 1}{8Oz^2} \tag{7.39}$$

The reduced pressure, $\bar{P}$, is plotted as a function of the Oz number for the three die casting machine performances as shown in Figure 7.10.

Figure 7.10 demonstrates that $\bar{P}$ monotonically decreases with an increase in the Oz number for all the machine performances. All the three results convert to the same line which is a plateau after $Oz = 20$. For large Oz numbers the reduced pressure, $\bar{P}$, can be considered to be constant $\bar{P} \simeq 0.025$. The gate velocity, in this case, is

$$U_3 \simeq 0.22 C_D \sqrt{\frac{P_{max}}{\rho}} \tag{7.40}$$

The Ozer number strongly depends on the discharge coefficient, C_D, and P_{max}. The value of Q_{max} is relatively insensitive to the size of the die casting machine. Thus, this equation is applicable to a well designed runner (large C_D) and/or a large die casting machine (large P_{max}).

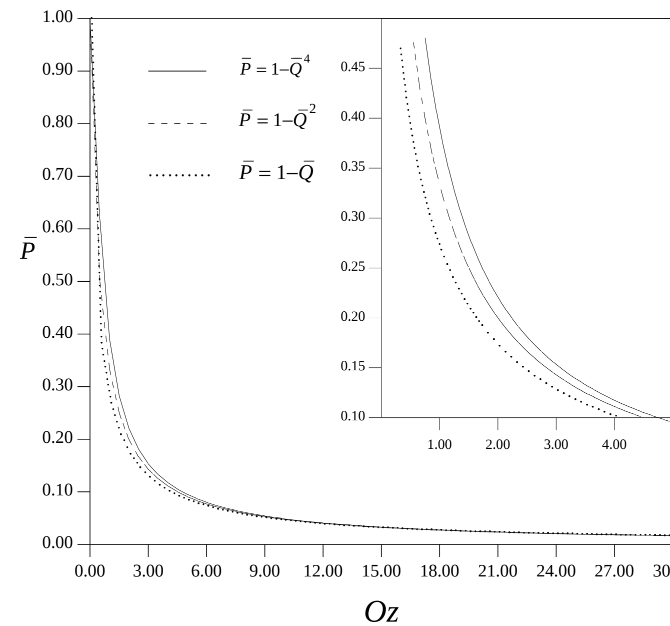


Fig. -7.10. Reduced pressure, $\bar{P}$, for various machine performances as a function of the Oz number.

The reduced pressure for a very small value of the Oz number equals to one, $\bar{P} \simeq 1$ or $P_{max} = P_1$, due to the large resistance in the runner (when the resistance in the runner approaches infinity, $K_F \to \infty$, then $\bar{P} = 1$). Hence, the gate velocity is determined by the approximation of

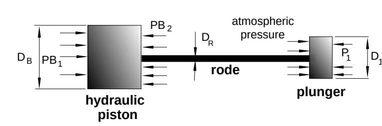


Fig. -7.11. Schematic of the plunger and piston balance forces.

$$U_3 \simeq C_D \sqrt{\frac{2P_{max}}{\rho}} \tag{7.41}$$

The difference between the various machine performances is more considerable in the middle range of the Oz numbers. A better machine performance produces a higher reduced pressure, $\bar{P}$. The preferred situation is when the Oz number is large and thus indicates that the machine performance is less important than the runner design parameters. This observation is further elucidated in view of Figures **??** and **??**.

Plunger area/diameter effects

The pressure at the plunger tip can be evaluated from a balance forces acts on the hydraulic piston and plunger as shown in Figure 7.11. The atmospheric pressure that acting on the left side of the plunger is neglected. Assuming a steady state and neglecting the friction, the forces balance on the rod yields

(why? perhaps to create a question for the students)

$$\frac{{D_B}^2\pi}{4}\left(P_{B1}-P_{B2}\right)+\frac{{D_R}^2\pi}{4}P_{B2}=\frac{{D_1}^2\pi}{4}P_1 \tag{7.42}$$

In particular, in the stationary case the maximum pressure obtains

$$\frac{{D_B}^2\pi}{4}\left.\left(P_{B1}-P_{B2}\right)\right|_{max}+\frac{{D_R}^2\pi}{4}\left.P_{B2}\right|_{max}=\frac{{D_1}^2\pi}{4}\left.P_1\right|_{max} \tag{7.43}$$

The equation (7.43) is reduced when the rode area is negligible; plus, notice that $\left.P_1\right|_{max}=P_{max}$ to read

$$\frac{{D_B}^2\pi}{4}\left.\left(P_{B1}-P_{B2}\right)\right|_{max}=\frac{{D_1}^2\pi}{4}P_{max} \tag{7.44}$$

Rearranging equation (7.44) yields

$$\left(\frac{D_B}{D_1}\right)^2=\frac{P_{max}}{\left.\left(P_{B1}-P_{B2}\right)\right|_{max}}\Longrightarrow P_{max}=\left.\left(P_{B1}-P_{B2}\right)\right|_{max}\left(\frac{D_B}{D_1}\right)^2 \tag{7.45}$$

20

The gate velocity relates to the liquid metal pressure at plunger tip according to the following equation combining equation (7.5) and (**??**) yields

$$U_3=C_D\sqrt{\frac{2}{\rho}}\sqrt{\frac{\left.\left(P_{B1}-P_{B2}\right)\right|_{max}\left(\frac{D_B}{D_1}\right)^2}{1+\frac{2}{\rho}\left(\frac{C_D\,A_3}{Q_{max}}\right)^2\left.\left(P_{B1}-P_{B2}\right)\right|_{max}\left(\frac{D_B}{D_1}\right)^2}} \tag{7.46}$$

Under the assumption that the machine characteristic is $P_1\propto\bar{Q}^2\Longrightarrow\bar{P}=1-\bar{Q}^2$,

---

[20] Note that $\left.P_1\right|_{max}\neq\left[P_1\right]_{max}$. The difference is that $\left.P_1\right|_{max}$ represents the maximum pressure of the liquid metal at plunger tip in the stationary case, where as $\left[P_1\right]_{max}$ represents the value of the maximum pressure of the liquid metal at the plunger tip that can be achieved when hydraulic pressure within the piston is varied. The former represents only the die casting machine and the shot sleeve, while the latter represents the combination of the die casting machine (and shot sleeve) and the runner system.

Equation (7.14) demonstrates that the value of $\left[P_1\right]_{max}$ is independent of P_{max} (for large values of P_{max}) under the assumptions in which this equation was attained (the "common" die casting machine performance, etc). This suggests that a smaller die casting machine can achieve the same job assuming average performance die casing machine.

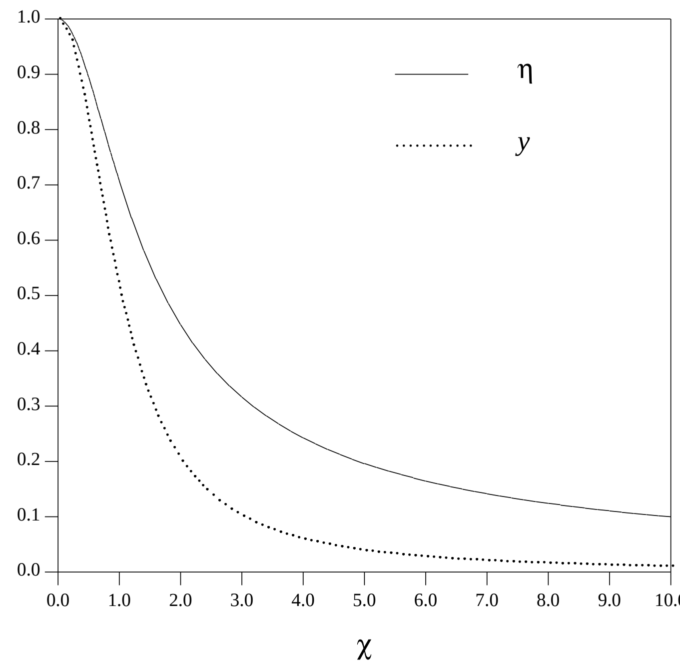


Fig. -7.12. Reduced liquid metal pressure at the plunger tip and reduced gate velocity as a function of the reduced plunger diameter.

Meta

the solution for the intersection point is given by equation ? To study equation (7.46), let's define

$$\chi = \sqrt{\frac{\rho}{(P_{B1} - P_{B2})|_{max}}} \left[\frac{Q_{max}}{C_D\, A_3}\right] \left[\frac{D_1}{D_B}\right] \tag{7.47}$$

and the reduced gate velocity

$$y = \frac{U_3\, A_3}{Q_{max}} \tag{7.48}$$

Using these definitions, equation (7.46) is converted to a simpler form:

$$y = \sqrt{\frac{1}{\chi^2 + 1}} \tag{7.49}$$

With these definitions, and denoting

$$\eta = P_1 \frac{2}{\rho} \left(\frac{C_D\, A_3}{Q_{max}}\right)^2 = 2\, Oz\, \bar{P} \tag{7.50}$$

one can obtain from equation (**??**) that (make a question about how to do it?)

$$\eta = \frac{1}{\chi^2 + 1} \tag{7.51}$$

The coefficients of P_1 in equation (7.50) and D_1 in equation (7.47) are assumed constant according to the "common" pQ^2 diagram. Thus, the plot of y and η as a function of χ represent the affect of the plunger diameter on the reduced gate velocity and reduced pressure. The gate velocity and the liquid metal pressure at plunger tip decreases with an increase in the plunger diameters, as shown in Figure 7.12 according to equations (7.49) and (7.51).

Meta End

A control volume as it is shown in Figure 7.13 is constructed to study the effect of the plunger diameter, (which includes the plunger with the rode, hydraulic piston, and shot sleeve, but which does not include the hydraulic liquid or the liquid metal jet). The control volume is stationary around the shot sleeve and is moving with the hydraulic piston. Applying the first law of thermodynamics, when that the atmospheric pressure is assumed negligible and neglecting the dissipation energy, yields

why? should be included in the end.

$$\dot{Q} + \dot{m}_{in}\left(h_{in} + \frac{{U_{in}}^2}{2}\right) = \dot{m}_{out}\left(h_{out} + \frac{{U_{out}}^2}{2}\right) + \left.\frac{dm}{dt}\right|_{c.v.}\left(e + \frac{{U_{c.v.}}^2}{2}\right) + \dot{W}_{c.v.} \tag{7.52}$$

In writing equation (7.52), it should be noticed that the only change in the control volume is in the shot sleeve. The heat transfer can be neglected, since the filling process is very rapid. There is no flow into the control volume (neglecting the air flow into the back side of the plunger and the change of kinetic energy of the air, why?), and therefore the second term on the right hand side can be omitted. Applying mass conservation on the control volume for the liquid metal yields

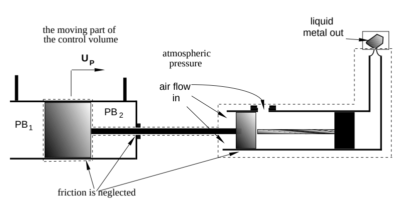


Fig. -7.13. A general schematic of the control volume of the hydraulic piston with the plunger and part of the liquid metal

$$\left.\frac{dm}{dt}\right|_{c.v.} = -\dot{m}_{out} \tag{7.53}$$

The boundary work on the control volume is done by the left hand side of the plunger and can be expressed by

$$\dot{W}_{c.v.} = -(P_{B1} - P_{B2})A_B U_1 \tag{7.54}$$

The mass flow rate out can be related to the gate velocity

$$\dot{m}_{out} = \rho A_3 U_3 \tag{7.55}$$

Mass conservation on the liquid metal in the shot sleeve and the runner yields

$$A_1 U_1 = A_3 \Longrightarrow {U_1}^2 = U_3^2 \left(\frac{A_3}{A_1}\right)^2 \tag{7.56}$$

Substituting equations (7.53-7.56) into equation (7.52) yields

$$(P_{B1} - P_{B2}) A_B U_3 A_1 = \rho A_3 U_3 \left[(h_{out} - e) + \frac{U_3^2}{2} \left(1 - \left(\frac{A_3}{A_1}\right)^2 \right) \right] \tag{7.57}$$

Rearranging equation (7.57) yields

$$(P_{B1} - P_{B2}) \frac{A_B}{A_1\,\rho} = (h_{out} - e) + \frac{U_3^2}{2} \left(1 - \left(\frac{A_3}{A_1}\right)^2 \right) \tag{7.58}$$

Solving for U_3 yields

$$U_3 = \sqrt{\frac{2\left[(P_{B1} - P_{B2}) \frac{A_B}{A_1\,\rho} - (h_{out} - e) \right]}{\left[1 - \left(\frac{A_3}{A_1}\right)^2 \right]}} \tag{7.59}$$

Or in term of the maximum values of the hydraulic piston

$$U_3 = \sqrt{\frac{2\left[\frac{(P_{B1} - P_{B2})|_{max}}{1+2\,Oz} \frac{A_B}{A_1\,\rho} - (h_{out} - e) \right]}{\left[1 - \left(\frac{A_3}{A_1}\right)^2 \right]}} \tag{7.60}$$

When the term $(h_{out} - e)$ is neglected ($C_p \sim C_v$ for liquid metal)

$$U_3 = \sqrt{\frac{2\, \frac{(P_{B1} - P_{B2})|_{max}}{1+2\,Oz} \frac{A_B}{A_1\,\rho}}{\left[1 - \left(\frac{A_3}{A_1}\right)^2 \right]}} \tag{7.61}$$

Normalizing the gate velocity equation (7.61) yields

$$y = \frac{U_3\,A_3}{Q_{max}} = \sqrt{\frac{C_D}{\chi^2 \left[1 + 2\,Oz\right] \left[1 - \left(\frac{A_3}{A_1}\right)^2 \right]}} \tag{7.62}$$

The expression (7.62) is a very complicated function of A_1. It can be shown that when the plunger diameter approaches infinity, $D_1 \rightarrow \infty$ (or when $A_1 \rightarrow \infty$) then the gate velocity approaches $U_3 \rightarrow 0$. Conversely, the gate velocity, $U_3 \rightarrow 0$, when the plunger diameter, $D_1 \rightarrow 0$. This occurs because mostly $K \rightarrow \infty$ and $C_D \rightarrow 0$. Thus, there is at least one plunger diameter that creates maximum velocity (see figure 7.14). A more detailed study shows that depending on the physics in the situation, more than one local maximum can occur. With a small plunger diameter, the gate velocity approaches zero because C_D approaches infinity. For a large plunger diameter, the gate velocity approaches zero because the pressure difference acting on the runner is approaching zero. The mathematical expression for the maximum gate velocity takes several pages, and therefore is not shown here. However, for practical purposes, the maximum velocity can easily (relatively) be calculated by using a computer program such as DiePerfect™.

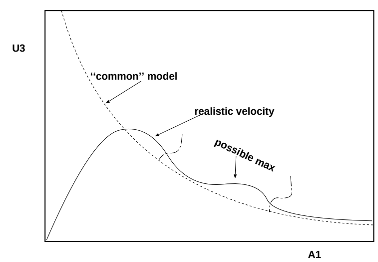


Fig. -7.14. The gate velocity, U_3 as a function of the plunger area, A_1

Machine size effect

The question how large the die casting machine depends on how efficient it is used. To maximized the utilization of the die casting machine we must understand under what condition it happens. It is important to realize that the injection of the liquid metal into the cavity requires power. The power, we can extract from a machine, depend on the plunger velocity and other parameters. We would like to design a process so that power extraction is maximized. Let's defined normalized machine size effect

$$\overline{pwrm} = \frac{\bar{Q}\Delta P}{P_{max} \times Q_{max}} \simeq \bar{Q} \times \bar{P} \tag{7.63}$$

Every die casting machine has a characteristic curve on the pQ^2 diagram as well. Assuming that the die casting machine has the "common" characteristic, $\bar{P} = 1 - \bar{Q}^2$, the normalized power can be expressed

$$\overline{pwrm} = \bar{Q}(1 - \bar{Q}^2) = \bar{Q}^2 - \bar{Q}^3 \tag{7.64}$$

where $\overline{pwrm}$ is the machine power normalized by $P_{max} \times Q_{max}$. The maximum power of this kind of machine is at 2/3 of the normalized flow rate, $\bar{Q}$, as shown in Figure **??**. It is recommended to design the process so the flow rate occurs at the vicinity of the maximum of the power. For a range of $1/3$ of $\bar{Q}$ that is from $0.5\bar{Q}$ to $0.83\bar{Q}$, the average power is 0.1388 $P_{max}Q_{max}$, as shown in Figure **??** by the shadowed rectangular. One may notice that this value is above the capability of the die casting machine in two ranges of the flow rate. The reason that this number is used is because with some

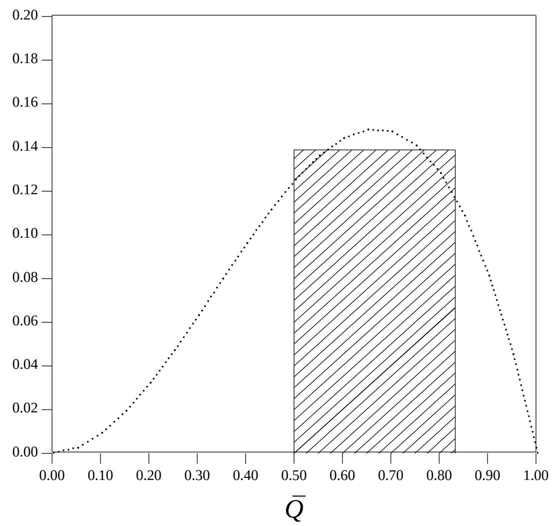


Fig. -7.15. The reduced power of the die casting machine as a function of the normalized flow rate.

improvements of the the runner design the job can be performed on this machine, and there is no need to move the job to a larger machine[21].

Precondition effect (wave formation)

Meta

discussion when $Q_1 \neq Q_3$

Meta End

7.4.3 Poor design effects

Meta

discussed the changes when different velocities are in different gates. Expanded on the sudden change to turbulent flow in one of the branches.

Meta End

7.4.4 Transient effects

Under construction

[21] Assuming that requirements on the clamping force is meet.

To put the discussion about the inertia of the system and compressibility. the magnitude analysis before intensification effects

insert only general remarks until the paper will submitted for publication

insert the notes from the yellow folder

7.5 Design Process

Now with these pieces of information how one design the process/runner system. A design engineer in a local company have told me that he can draw very quickly the design for the mold and start doing the experiments until he gets the products running well. Well, the important part should not be how quickly you get it to try on your machine but rather how quickly you can produce a good quality product and how cheap (little scrap as possible and smaller die casting machine). Money is the most important factor in the production. This design process is longer than just drawing the runner and it requires some work. However, getting the production going is much more faster in most cases and cheaper (less design and undesign scrap and less experiments/starting cost). Hence, for given die geometry, four conditions (actually there are more) need to satisfied

$$\frac{\partial U_3}{\partial A_1} = 0 \tag{7.65}$$

$$\frac{\partial U_3}{\partial A_3} = 0 \tag{7.66}$$

the clamping force, and satisfy the power requirements.

For these criteria the designer has to check the runner design to see if gate velocity are around the recommended range. A possible answer has to come from financial considerations, since we are in the business of die casting to make money. Hence, the optimum diameter is the one which will cost the least (the minimum cost). How, then, does the plunger size determine cost? It has been shown that plunger diameter has a value where maximum gate velocity is created.

General relationship between runner hydraulic diameter and plunger diameter.

A very large diameter requires a very large die casting machine (due to physical size and the weight of the plunger). So, one has to chose as first approximation the largest plunger on a smallest die casting machine. Another factor has to be taken into consideration is the scrap created in the shot sleeve. Obviously, the liquid metal in the sleeve has to be the last place to solidify. This requires the biscuit to be of at least the same thickness as the runner.

$$T_{runner} = T_{biscuit} \tag{7.67}$$

Therefore, the scrap volume should be

$$\frac{\pi {D_1}^2}{4} T_{biscuit} \Longrightarrow \frac{\pi {D_1}^2}{4} T_{runner} \tag{7.68}$$

When the scrap in the shot sleeve becomes significant, compared to scrap of the runner

$$\frac{\pi {D_1}^2}{4} T_{runner} = \bar{L} T_{runner} \tag{7.69}$$

Thus, the plunger diameter has to be in the range of

$$D_1 = \sqrt{\frac{4}{\pi}\bar{L}} \tag{7.70}$$

To discussed that the plunger diameter should not be use as varying the plunger diameter to determine the gate velocity

7.6 The Intensification Consideration

Intensification is a process in which pressure is increased making the liquid metal flows during the solidification process to ensure compensation for the solidification shrinkage of the liquid metal (up to 20%). The intensification is applied by two methods: one) by applying additional pump, two) by increasing the area of the actuator (the multiplier method, or the prefill method)[22].

put schematic figure of how it is done from the patent by die casting companies

. The first method does not increase the intensification force to "P_{max}" by much. However, the second method, commonly used today in the industry, can increase considerably the ratio.

Meta

Analysis of the forces demonstrates that as first approximation the plunger diameter does not contribute any additional force toward pushing the liquid metal.

Meta End

why? to put discussion

A very small plunger diameter creates faster solidification, and therefore the actual force is reduced. Conversely, a very large plunger diameter creates a very small pressure for driving the liquid metal.

discuss the the resistance as a function of the diameter

7.7 Summary

In this chapter it has been shown that the "common" diagram is not valid and produces unrealistic trends therefore has no value what so ever[23]. The reformed pQ^2 diagram was introduced. The mathematical theory/presentation based on established scientific principles was introduced. The effects of various important parameters was discussed. The method of designing the die casting process was discussed. The plunger diameter

[22] A note for the manufactures, if you would like to have your system described here with its advantages, please drop me a line and I will discuss with you about the material that I need. I will not charge you any money.

[23] Beside the historical value

has a value for which the gate velocity has a maximum. For $D_1 \rightarrow 0$ gate velocity, $U_3 \rightarrow 0$ when $D_1 \rightarrow \infty$ the same happen $U_3 \rightarrow 0$. Thus, this maximum gate velocity determines whether an increase in the plunger diameter will result in an increase in the gate velocity or not. An alternative way has been proposed to determine the plunger diameter.

7.8 Questions

Garber concluded that his model was not able to predict an acceptable value for critical velocity for fill percentages lower than 50% . . .

Brevick, Ohio

CHAPTER 8

Critical Slow Plunger Velocity

8.1 Introduction

This Chapter deals with the first stage of the injection in a cold chamber machine in which the desire (mostly) is to expel maximum air/gas from the shot sleeve. Porosity is a major production problem in which air/gas porosity constitutes a large portion. Minimization of Air Entrainment in the Shot Sleeve (AESS) is a prerequisite for reducing air/gas porosity. This can be achieved by moving the plunger at a specific speed also known as the critical slow plunger velocity. It happens that this issue is related to the hydraulic jump, which was discussed in the previous Chapters 5 (accidentally? you thought!).

The "common" model, also known as Garber's model, with its extensions made by Brevick[1], Miller[2], and EKK's model are presented first here. The basic fundamental errors of these models are presented. Later, the reformed and "simple" model is described. It followed by the transient and poor design effects[3]. Afterwords, as usual questions are given at the end of the chapter.

8.2 The "common" models

In this section the "common" models are described. Since the "popular" model also known as Garber's model never work (even by its own creator)[4], several other models have appeared. These models are described here to have a clearer picture of what

[1] Industrial and Systems Engineering (ISE) Graduate Studies Chair, ISE department at The Ohio State University

[2] The chair of ISE Dept. at OSU

[3] It be added in the next addition

[4] I wonder if Garber and later Brevick have ever considered that their the models were simply totally false.

was in the pre Bar–Meir's model. First, a description of Garber's model is given later Brevick's two models along with Miller's model[5] are described briefly. Lastly, the EKK's numerical model is described.

8.2.1 Garber's model

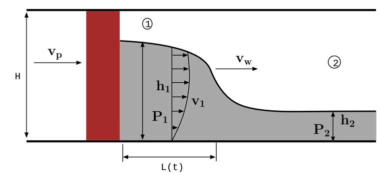


Fig. -8.1. A schematic of wave formation in stationary coordinates

The description in this section is based on one of the most cited paper in the die casting research [17]. Garber's model deals only with a plug flow in a circular cross–section. In this section, we "improve" the model to include any geometry cross section with any velocity profile[6].

Consider a duct (any cross section) with a liquid at level h_2 and a plunger moving from the left to the right, as shown in Figure 8.1. Assuming a quasi steady flow is established after a very short period of time, a unique height, h_1, and a unique wave velocity, V_w, for a given constant plunger velocity, V_p are created. The liquid in the substrate ahead of the wave is still, its height, h_2, is determined by the initial fill. Once the height, h_1, exceeds the height of the shot sleeve, H, there will be splashing. The splashing occurs because no equilibrium can be achieved (see Figure 8.2a). For h_1 smaller than H, a reflecting wave from the opposite wall appears resulting in an enhanced air entrainment (see Figure 8.2b). Thus, the preferred situation is when $h_1 = H$ (in circular shape $H = 2R$) in which case no splashing or a reflecting wave result.

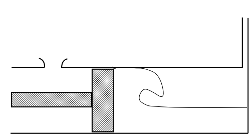


Fig a. *A schematic of built wave.*

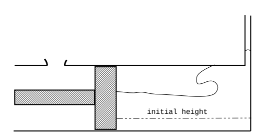


Fig b. *A schematic of reflecting wave.*

Fig. -8.2. The left graph depicts the "common" pQ^2 version. The right graph depicts P_{max} and Q_{max} as a function of the plunger diameter according to "common" model.

It is easier to model the wave with coordinates that move at the wave velocity, as shown in Figure 8.3. With the moving coordinate, the wave is stationary, the plunger moves back at a velocity $(V_w - V_p)$, and the liquid moves from the right to the left. Dashed line shows the stationary control volume.

[5]This model was developed at Ohio State University by Miller's Group in the early 1990's.

[6]This addition to the original Garber's paper is derived here. I assumed that in this case, some mathematics will not hurt the presentation.

Mass conservation of the liquid in the control volume reads:

$$\int_{A_2} \rho V_w dA = \int_{A_1} \rho(v_1 - V_p) dA \quad (8.1)$$

where v_1 is the local velocity. Under quasi-steady conditions, the corresponding average velocity equals the plunger velocity:

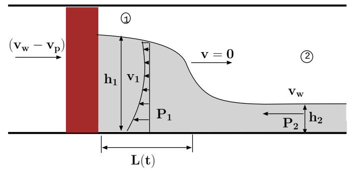


Fig. -8.3. A schematic of the wave with moving coordinates

$$\frac{1}{A_1}\int_{A_1} v_1 dA = \overline{v_1} = V_p \tag{8.2}$$

What is justification for equation 8.2? Assuming that heat transfer can be neglected because of the short process duration[7]. Therefore, the liquid metal density (which is a function of temperature) can be assumed to be constant. Under the above assumptions, equation (8.1) can be simplified to

build a question about what happens if the temperature changes by a few degrees. How much will it affect equation 8.2 and other parameters?

$$V_w A(h_2) = (V_w - V_p) A(h_1) \quad ; \quad A(h_i) = \int_0^{h_i} dA \tag{8.3}$$

Where i in this case can take the value of 1 or 2. Thus,

$$\frac{V_w}{(V_w - V_p)} = f(h_{12}) \tag{8.4}$$

where $f(h_{12}) = \frac{A(h_1)}{A(h_2)}$ is a dimensionless function. Equation (8.4) can be transformed into a dimensionless form:

$$f(h_{12}) = \frac{\tilde{v}}{(\tilde{v} - 1)} \tag{8.5}$$

$$\Longrightarrow \tilde{v} = \frac{f(h_{12})}{f(h_{12}) - 1} \tag{8.6}$$

where $\tilde{v} = \frac{V_w}{V_p}$. Show that $A(h_1) = 2\pi R^2$ for $h_1 = 2R$ Assuming energy is conserved (the Garber's model assumption), and under conditions of negligible heat transfer, the energy conservation equation for the liquid in the control volume (see Figure 8.3) reads:

$$\int_{A_1} \left[\frac{P_{\mathsf{B}}}{\rho} + \frac{\gamma_E (V_w - V_p)^2}{2}\right](V_w - V_p) dA = \int_{A_2} \left[\frac{P_2}{\rho} + \frac{{V_w}^2}{2}\right] V_w dA \tag{8.7}$$

where

$$\gamma = \frac{1}{A_1 (V_w - V_p)^3}\int_{A_1} (V_w - v_1)^3 dA = \frac{1}{A_1(\tilde{v} - 1)^3}\int_{A_1}\left(\tilde{v} - \frac{v_1}{V_p}\right)^3 dA \tag{8.8}$$

[7]see Chapter 3 for a detailed discussion

under–construction The shape factor, γ_E, is introduced to account for possible deviations of the velocity profile at section 1 from a pure plug flow. Note that in die casting, the flow is pushed by the plunger and can be considered as an inlet flow into a duct. The typical Re number is 10^5, and for this value the entry length is greater than 50m, which is larger than any shot sleeve by at least two orders of magnitude.

The pressure in the gas phase can be assumed to be constant. The hydrostatic pressure in the liquid can be represent by $R\bar{y}_c g\rho$ [28], where $R\bar{y}_c$ is the center of the cross section area. For a constant liquid density equation (8.7) can be rewritten as:

$$\left[R\bar{y}_{c1}g + \gamma_E \frac{(V_w - V_p)^2}{2}\right](V_w - V_p)A(h_1) = \left[R\bar{y}_{c2}g + \frac{{V_w}^2}{2}\right] V_w A(h_2) \tag{8.9}$$

Garber (and later Brevick) put this equation plus several geometrical relationships as the solution. Here we continue to obtain an analytical solution. Defining a dimensionless parameter Fr as

$$Fr = \frac{Rg}{{V_p}^2}, \tag{8.10}$$

Utilizing definition (8.10) and rearranging equation (8.9) yields

$$2Fr_E \times \bar{y}_{c1} + \gamma_E(\tilde{v} - 1)^2 = 2Fr_E \times \bar{y}_{c2} + \tilde{v}^2 \tag{8.11}$$

Solving equation (8.11) for Fr_E the latter can be further rearranged to yield:

$$Fr_E = \sqrt{\frac{2(\bar{y}_{c1} - \bar{y}_{c2})}{\frac{(1+\gamma_E)f(h_{12})}{f(h_{12})-1} - \gamma_E}} \tag{8.12}$$

Given the substrate height, equation (8.12) can be evaluated for the Fr_E, and the corresponding plunger velocity ,V_p. which is defined by equation (8.10). This solution will be referred herein as the "energy solution".

8.2.2 Brevick's Model

The square shot sleeve

Since Garber's model never work Brevick and co–workers go on a "fishing expedition" in the fluid mechanics literature to find equations to describe the wave. They found in Lamb's book several equations relating the wave velocity to the wave height for a deep liquid (water)[8]. Since these equations are for a two dimensional case, Brevick and co–workers built it for a squared shot sleeve. Here are the equations that they used. The "instantaneous" height difference ($\Delta h = h_1 - h_2$) is given as

$$\Delta h = h_2 \left[\frac{V_p}{2\sqrt{gh_2}} + 1\right]^2 - h_2 \tag{8.13}$$

[8]I have checked the reference and I still puzzled by the equations they found?

This equation (8.13), with little rearranging, obtained a new form

$$V_p = 2\sqrt{gh_2}\left[\sqrt{\frac{h_1}{h_2}} - 1\right] \tag{8.14}$$

The wave velocity is given by

$$V_w = \sqrt{gh_2}\left[3\sqrt{1 + \frac{\Delta h}{h_2}} - 2\right] \tag{8.15}$$

Brevick introduces the optimal plunger acceleration concept. "By plotting the height and position of each incremental wave with time, their model is able to predict the 'stability' of the resulting wave front when the top of the front has traveled the length of the shot sleeve."[9]. They then performed experiments on this "miracle acceleration [10]."

8.2.3 Brevick's circular model

Probably, because it was clear to the authors that the previous model was only good for a square shot sleeve [11]. They say let reuse Garber's model for every short time steps and with different velocity (acceleration).

8.2.4 Miller's square model

Miller and his student borrowed a two dimensional model under assumption of turbulent flow. They assumed that the flow is "infinite" turbulence and therefor it is a plug flow[12]. Since the solution was for 2D they naturally build model for a square shot sleeve[13]. The mass balance for square shot sleeve

$$V_w h_2 = (V_w - V_p)h_1 \tag{8.16}$$

Momentum balance on the same control volume yield

$$\left[\frac{P_{\mathsf{B}}}{\rho} + \frac{(V_w - V_p)^2}{2}\right](V_w - V_p)h_1 = \left[\frac{P_2}{\rho} + \frac{{V_w}^2}{2}\right]V_w h_2 \tag{8.17}$$

and the solution of these two equations is

$$Fr_{miller} = \frac{1}{2}\frac{h_1}{h_2}\left(\frac{h_1}{h_2} + 1\right) \tag{8.18}$$

[9] What an interesting idea?? Any physics?

[10] As to say this is not good enough a fun idea, they also "invented" a new acceleration units "cm/sec–cm".

[11] It is not clear whether they know that this equations are not applicable even for a square shot sleeve.

[12] How they come–out with this conclusion?

[13] Why are these two groups from the same university and the same department not familiar with each others work.

8.3 The validity of the "common" models

8.3.1 Garber's model

Energy is known to dissipate in a hydraulic jump in which case the equal sign in equation (8.12) does not apply and the criterion for a nonsplashing operation would read

$$Fr_E < Fr_{optimal} \tag{8.19}$$

A considerable amount of research work has been carried out on this wave, which is known in the scientific literature as the hydraulic jump. The hydraulic jump phenomenon has been studied for the past 200 years. Unfortunately, Garber, (and later other researchers in die casting – such as Brevick and his students from Ohio State University [8], [31])[14], ignored the previous research. This is the real reason that their model never works. Show the relative error created by Garber's model when the substrate height h_2 is the varying parameter.

8.3.2 Brevick's models

square model

There are two basic mistakes in this model, first) the basic equations are not applicable to the shot sleeve situation, second) the square geometry is not found in the industry. To illustrate why the equations Brevick chose are not valid, take the case where $1 > h_1/h_2 > 4/9$. For that case V_w is positive and yet the hydraulic jump opposite to reality ($h_1 < h_2$).

Improved Garber's model

Since Garber's model is scientific erroneous any derivative that is based on it no better than its foundation[15].

8.3.3 Miller's model

The flow in the shot sleeve in not turbulent[16]. The flow is a plug flow because entry length problem[17].

Besides all this, the geometry of the shot sleeve is circular. This mistake is discussed in the comparison in the discussion section of this chapter.

[14] Even with these major mistakes NADCA under the leadership of Gary Pribyl and Steve Udvardy continues to award Mr. Brevick with additional grands to continue this research until now, Why?

[15] I wonder how much NADCA paid Brevick for this research?

[16] Unless someone can explain and/or prove otherwise.

[17] see Chapter 3.

8.3.4 EKK's model (numerical model)

This model based on numerical simulations based on the following assumptions: 1) the flow is turbulent, 2) turbulence was assume to be isentropic homogeneous every where ($k\epsilon$ model), 3) un–specified boundary conditions at the free interface (how they solve it with this kind of condition?), and 4) unclear how they dealt with the "corner point" in which plunger perimeter in which smart way is required to deal with zero velocity of the sleeve and known velocity of plunger.

Several other assumptions implicitly are in that work[18] such as no heat transfer, a constant pressure in the sleeve etc.

According to their calculation a jet exist somewhere in the flow field. They use the $k\epsilon$ model for a field with zero velocity! They claim that they found that the critical velocity to be the same as in Garber's model. The researchers have found same results regardless the model used, turbulent and laminar flow!! One can only wonder if the usage of $k\epsilon$ model (even for zero velocity field) was enough to produce these erroneous results or perhaps the problem lays within the code itself[19].

8.4 *The Reformed Model*

The hydraulic jump appears in steady–state and unsteady–state situations. The hydraulic jump also appears when using different cross–sections, such as square, circular, and trapezoidal shapes. The hydraulic jump can be moving or stationary. The "wave" in the shot sleeve is a moving hydraulic jump in a circular cross–section. For this analysis, it does not matter if the jump is moving or not. The most important thing to understand is that a large portion of the energy is lost and that this cannot be neglected. All the fluid mechanics books[20] show that Garber's formulation is not acceptable and a different approach has to be employed. Today, the solution is available to die casters in a form of a computer program – DiePerfect™.

8.4.1 The reformed model

In this section the momentum conservation principle is applied on the control volume in Figure 8.3. For large Re ($\sim 10^5$) the wall shear stress can be neglected compared to the inertial terms (the wave is assumed to have a negligible length). The momentum balance reads:

$$\int_{A_1} \left[P_\mathsf{B} + \rho\gamma_M (V_w - V_p)^2\right] dA = \int_{A_2} \left[P_2 + \rho {V_w}^2\right] dA \tag{8.20}$$

where

$$\gamma_M = \frac{1}{A_1 (V_w - V_p)^2} \int_{A_1} (V_w - v_1)^2 dA = \frac{1}{A_1 (\tilde{v} - 1)^2} \int_{A_1} \left(\tilde{v} - \frac{v_1}{V_p}\right)^2 dA \tag{8.21}$$

[18] This paper is a good example of poor research related to a poor presentation and text processing.
[19] see remark on page 44
[20] in the last 100 years

Given the velocity profile v_1, the shape factor γ_M can be obtained in terms of $\tilde{v}$. The expressions for γ_M for laminar and turbulent velocity profiles at section 1 easily can be calculated. Based on the assumptions used in the previous section, equation (8.20) reads:

$$\left[R\bar{y}_{c1}g + \gamma_M(V_w - V_p)^2\right] A(h_1) = \left[R\bar{y}_{c2}g + {V_w}^2\right] A(h_2) \tag{8.22}$$

Rearranging equation (8.22) into a dimensionless form yields:

$$f(h_{12})\left[\bar{y}_{c1}Fr + \gamma_M(\tilde{v}-1)^2\right] = \bar{y}_{c2}Fr + \tilde{v}^2 \tag{8.23}$$

Combining equations (8.5) and (8.23) yields

$$Fr_M = \frac{\left[\frac{f(h_{12})}{f(h_{12})-1}\right]^2 - \gamma_M\, f(h_{12})\left[\left(\frac{f(h_{12})}{f(h_{12})-1}\right) - 1\right]^2}{\left[f(h_{12})\bar{y}_{c1} - \bar{y}_{c2}\right]} \tag{8.24}$$

where Fr_M is the Fr number which evolves from the momentum conservation equation. Equation (8.24) is the analogue of equation (8.12) and will be referred herein as the "Bar–Meir's solution".

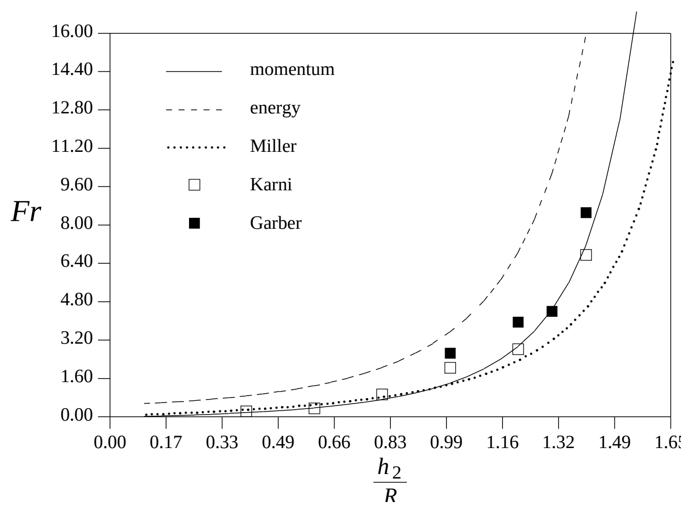


Fig. -8.4. The Froude number as a function of the relative height.

[21] and the "energy solution" can be presented in a simple form. Moreover, these solutions can be applied to any cross section for the transition of the free surface flow to pressurized flow. The discussion here focuses on the circular cross section, since it is the only one used by diecasters. Solutions for other velocity profiles, such as laminar

[21] This model was constructed with a cooperation of a another researcher.

flow (Poiseuille paraboloid), are discussed in the Appendix [22]. Note that the Froude number is based on the plunger velocity and not on the upstream velocity commonly used in the two–dimensional hydraulic jump.

The experimental data obtained by Garber , and Karni and the transition from the free surface flow to pressurized flow represented by equations (8.12) and (8.24) for a circular cross section are presented in Figure 8.4 for a plug flow. The Miller's model (two dimensional) of the hydraulic jump is also presented in Figure 8.4. This Figure shows clearly that the "Bar–Meir's solution" is in agreement with Karni's experimental results. The agreement between Garber's experimental results and the "Bar–Meir's solution," with the exception of one point (at $h_2 = R$), is good.

The experimental results obtained by Karni were taken when the critical velocity was obtained (liquid reached the pipe crown) while the experimental results from Garber are interpretation (kind of average) of subcritical velocities and supercritical velocities with the exception of the one point at $h_2/R = 1.3$ (which is very closed to the "Bar–Meir's solution"). Hence, it is reasonable to assume that the accuracy of Karni's results is better than Garber's results. However, these data points have to be taken with some caution[23]. Non of the experimental data sets were checked if a steady state was achieved and it is not clear how the measurements carried out.

It is widely accepted that in the two dimensional hydraulic jump small and large eddies are created which are responsible for the large energy dissipation [19]. Therefore, energy conservation cannot be used to describe the hydraulic jump heights. The same can be said for the hydraulic jump in different geometries. Of course, the same has to be said for the circular cross section. Thus, the plunger velocity has to be greater than the one obtained by Garber's model, which can be observed in Figure 8.4. The Froude number for the Garber's model is larger than the Froude number obtained in the experimental results. Froude number inversely proportional to square of the plunger velocity, $Fr \propto 1/{V_p}^2$ and hence the velocity is smaller. The Garber's model therefore underestimates the plunger velocity.

8.4.2 Design process

To obtain the critical slow plunger velocity, one has to follow this procedure:

1. Calculate/estimate the weight of the liquid metal.
2. Calculate the volume of the liquid metal (make sure that you use the liquid phase property and not the solid phase).
3. Calculate the percentage of filling in the shot sleeve, $\frac{height}{r}$.
4. Find the Fr number from Figure 8.4.
5. Use the Fr number found to calculate the plunger velocity by using equation (8.10).

[22] To appear in the next addition.
[23] Results of good experiments performed by serious researchers are welcome.

8.5 Summary

In this Chapter we analyzed the flow in the shot sleeve and developed a explicit expression to calculated the required plunger velocity. It has been shown that Garber's model is totally wrong and therefore Brevick's model is necessarily erroneous as well. The same can be said to all the other models discussed in this Chapter. The connection between the "wave" and the hydraulic jump has been explained. The method for calculating the critical slow plunger velocity has been provided.

8.6 Questions

CHAPTER 9

Venting System Design

The difference between the two is expressed by changing standard atmospheric ambient conditions to those existing in the vacuum tank.

Miller's student, p. 102

9.1 Introduction

Proper design of the venting system is one of the requirements for reducing air/gas porosity. Porosity due to entrainment of gases constitutes a large portion of the total porosity, especially when the cast walls are very thin (see Figure **??**). The main causes of air/gas porosity are insufficient vent area, lubricant evaporation (reaction processes), incorrect placement of the vents, and the mixing processes. The present chapter considers the influence of the vent area (in atmospheric and vacuum venting) on the residual gas (in the die) at the end of the filling process.

Atmospheric venting, the most widely used casting method, is one in which the vent is opened to the atmosphere and is referred herein as air venting. Only in extreme cases are other solutions required, such as vacuum venting, Pore Free Technique (in zinc and aluminum casting) and squeeze casting. Vacuum is applied to extract air/gas from the mold before it has the opportunity to mix with the liquid metal and it is call vacuum venting. The Pore Free technique is a variation of the vacuum venting in which the oxygen is introduced into the cavity to replace the air and to react with the liquid metal, and therefore creates a vacuum [5]. Squeeze

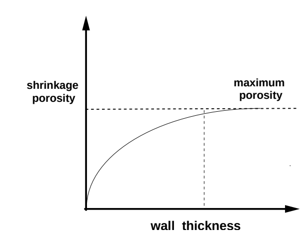


Fig. -9.1. The relative shrinkage porosity as a function of the casting thickness.

casting is a different approach in which the surface tension is increased to reduce the possible mixing processes (smaller Re number as well). The gases in the shot sleeve and cavity are made mostly of air and therefore the term "air" is used hereafter. These three "solutions" are cumbersome and create a far more expensive process. In this chapter, a qualitative discussion on when these solutions should be used and when they are not needed is presented.

Obviously, the best ventilation is achieved when a relatively large vent area is designed. However, to minimize the secondary machining (such as trimming), to ensure freezing within the venting system, and to ensure breakage outside the cast mold, vents have to be very narrow. A typical size of vent thicknesses range from 1–2$[mm]$. These conflicting requirements on the vent area suggest an optimum area. As usual the "common" approach is described the errors are presented and the reformed model is described.

9.2 The "common" models

9.2.1 Early (etc.) model

The first model dealing with the extraction of air from the cavity was done by Sachs. In this model, Sachs developed a model for the gas flow from a die cavity based on the following assumptions: 1) the gas undergoes an isentropic process in the die cavity, 2) a quasi steady state exists, 3) the only resistance to the gas flow is at the entrance of the vent, 4) a "maximum mass flow rate is present", and 5) the liquid metal has no surface tension, thus the metal pressure is equal to the gas pressure. Sachs also differentiated between two cases: choked flow and un–choked flow (but this differentiation did not come into play in his model). Assumption 3 requires that for choked flow the pressure ratio be about two between the cavity and vent exit.

Almost the same model was repeat by several researchers[1]. All these models, with the exception of Veinik , neglect the friction in the venting system. The vent design in a commercial system includes at least an exit, several ducts, and several abrupt expansions/contractions in which the resistance coefficient ($\frac{4fL}{D}$ see [29, page 163]) can be evaluated to be larger than 3 and a typical value of $\frac{4fL}{D}$ is about 7 or more. In this case, the pressure ratio for the choking condition is at least 3 and the pressure ratio reaches this value only after about 2/3 of the piston stroke is elapsed. It can be shown that when the flow is choked the pressure in the cavity does not remain constant as assumed in the models but increases exponentially.

9.2.2 Miller's model

Miller and his student, in the early 90's, constructed a model to account for the friction in the venting system. They based their model on the following assumptions:

1. No heat transfer

[1] Apparently, no literature survey was required/available/needed at that time.

2. Isothermal flow (constant temperature) in the entrance to vent (according to the authors in the presentation)

3. Fanno flow in the rest vent

4. Air/gas obeys the ideal gas model

Miller and his student described the calculation procedures for the two case as choked and unchoked conditions. The calculations for the choked case are standard and can be found in any book about Fanno flow but with an interesting twist. The conditions in the mold and the sleeve are calculated according the ambient condition (see the smart quote of this Chapter)[2]. The calculations about unchoked case are very interesting and will be discussed here in a little more details. The calculations procedure for the unchoked as the following:

- Assume M_{in} number (entrance Mach number to the vent) lower than M_{in} for choked condition
- Calculate the corresponding star (choked conditions) $\frac{4fL}{D}$, the pressure ratio, and the temperature ratio for the assume M_{in} number
- Calculate the difference between the calculated $\frac{4fL}{D}$ and the actual $\frac{4fL}{D}$.
- Use the difference $\frac{4fL}{D}$ to calculate the double stars (theoretical exit) conditions based on the ambient conditions.
- Calculated the conditions in the die based on the double star conditions.

Now the mass flow out is determined by mass conservation.

Of course, these calculations are erroneous. In choked flow, the conditions are determined **only and only** by up–steam and never by the down steam[3]. The calculations for unchoked flow are mathematical wrong. The assumption made in the first step never was checked. And mathematically speaking, it is equivalent to just guessing solution. These errors are only fraction of the other other in that model which include among other the following: one) assumption of constant temperature in the die is wrong, two) poor assumption of the isothermal flow, three) poor measurements etc. On top of that was is the criterion for required vent area.

9.3 General Discussion

When a incompressible liquid such as water is pushed, the same amount propelled by the plunger will flow out of the system. However, air is a compressible substance and thus the above statement cannot be applied. The flow rate out depends on the resistance to the flow plus the piston velocity (piston area as well). There could be three situations

[2]This model results in negative temperature in the shot sleeve in typical range.

[3]How otherwise, can it be? It is like assuming negative temperature in the die cavity during the injection. Is it realistic?

1) the flow rate out is **less** than the volume pushed by the piston, 2) the flow rate out is **more** than the volume pushed by the piston, or 3) the flow rate out is **equal** to the volume pushed by the piston. The last case is called the critical design, and it is associated with the critical area.

Air flows in the venting system can reach very large velocities up to about 350 $[m/sec]$. The air cannot exceed this velocity without going through a specially configured conduit (converging diverging conduit). This phenomena is known by the name of "choked flow". This physical phenomenon is the key to understanding the venting design process. In air venting, the venting system has to be designed so that air velocity does not reach the speed of sound: in other words, **the flow is not choked**. In vacuum venting, the air velocity reaches the speed of sound almost instantaneously, and the design should be such that it ensures that the air pressure does not exceed the atmospheric pressure.

Prior models for predicting the optimum vent area did not consider the resistance in the venting system (pressure ratio of less than 2). The vent design in a commercial system includes at least an exit, several ducts, and several abrupt expansions/contractions in which the resistance coefficient, $\frac{4fL}{D}$, is of the order of 3–7 or more. Thus, the pressure ratio creating choked flow is at least 3. One of the differences between vacuum venting and atmospheric venting occurs during the start–up time. For vacuum venting, a choking condition is established almost instantaneously (it depends on the air volume in the venting duct), while in the atmospheric case the volume of the air has to be reduced to more than half (depending on the pressure ratio) before the choking condition develops - - and this can happen only when more than 2/3 or more of the piston stroke is elapsed. Moreover, the flow is not necessarily choked in atmospheric venting. Once the flow is choked, there is no difference in calculating the flow between these two cases. It turns out that the mathematics in both cases are similar, and therefore both cases are presented in the present chapter.

The role of the chemical reactions was shown to be insignificant. The difference in the gas solubility (mostly hydrogen) in liquid and solid can be shown to be insignificant [1]. For example, the maximum hydrogen release during solidification of a kilogram of aluminum is about $7cm^3$ at atmospheric temperature and pressure. This is less than 3% of the volume needed to be displaced, and can be neglected. Some of the oxygen is depleted during the filling time [5]. The last two effects tend to cancel each other out, and the net effect is minimal.

The numerical simulations produce unrealistic results and there is no other quantitative tools for finding the vent locations (the last place(s) to be filled) and this issue is still an open question today. There are, however, qualitative explanations and reasonable guesses that can push the accuracy of the last place (the liquid metal reaches) estimate to be within the last 10%–30% of the filling process. This information increases the significance of the understanding of what is the required vent area. Since most of the air has to be vented during the initial stages of the filling process, in which the vent locations do not play a role.

Air venting is the cheapest method of operation, and it should be used unless acceptable results cannot be obtained using it. Acceptable results are difficult to obtain

1) when the resistance to the air flow in the mold is more significant than the resistance in the venting system, and 2) when the mixing processes are augmented by the specific mold geometry. In these cases, the extraction of the air prior to the filling can reduce the air porosity which require the use of other techniques.

An additional objective is to provide a tool to "combine" the actual vent area with the resistance (in the venting system) to the air flow; thus, eliminating the need for calculations of the gas flow in the vent in order to minimize the numerical calculations. Hu et al. and others have shown that the air pressure is practically uniform in the system. Hence, this analysis can also provide the average air pressure that should be used in numerical simulations.

9.4 The Analysis

The model is presented here with a minimal of mathematical details. However, emphasis is given to all the physical understanding of the phenomena. The interested reader can find more detailed discussions in several other sources [4]. As before, the integral approach is employed. All the assumptions which are used in this model are stated so that they can be examined and discussed at the conclusion of the present chapter. Here is a list of the assumptions which are used in developing this model:

1. The main resistance to the air flow is assumed to be in the venting system.
2. The air flow in the cylinder is assumed one–dimensional.
3. The air in the cylinder undergoes an isentropic process.
4. The air obeys the ideal gas model, $P = \rho R T$.
5. The geometry of the venting system does not change during the filling process (i.e., the gap between the plates does not increase during the filling process).
6. The plunger moves at a constant velocity during the filling process, and it is determined by the pQ^2 diagram calculations.
7. The volume of the venting system is negligible compared to the cylinder volume.
8. The venting system can be represented by one long, straight conduit.
9. The resistance to the liquid metal flow, $\frac{4fL}{D}$, does not change during the filling process (due to the change in the Re, or Mach numbers).
10. The flow in the venting system is an adiabatic flow (Fanno flow).
11. The resistance to the flow, $\frac{4fL}{D}$, is not affected by the change in the vent area.

With the above assumptions, the following model as shown in Figure **??** is proposed. A plunger pushes the liquid metal, and both of them (now called as the piston) propel the air through a long, straight conduit.

The mass balance of the air in the cylinder yields

$$\frac{dm}{dt} + \dot{m}_{out} = 0. \tag{9.1}$$

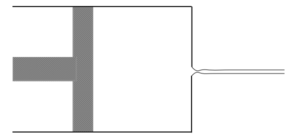


Fig. -9.2. A simplified model for the venting system.

This equation (9.1) is the only equation that needed to be solved. To solve it, the physical properties of the air need to be related to the geometry and the process. According to assumption 4, the air mass can be expressed as

$$m = \frac{PV}{RT} \tag{9.2}$$

The volume of the cylinder under assumption 6 can be written as

$$\frac{V(t)}{V(0)} = \left(1 - \frac{t}{t_{max}}\right) \tag{9.3}$$

Thus, the first term in equation (9.1) is represented by

$$\frac{dm}{dt} = \frac{d}{dt}\left(\frac{PV(0)\left(1 - \frac{t}{t_{max}}\right)}{RT}\right) \tag{9.4}$$

The filling process occurs within a very short period time [$milliseconds$], and therefore the heat transfer is insignificant 3. This kind of flow is referred to as Fanno flow[4]. The instantaneous flow rate has to be expressed in terms of the resistance to the flow, $\frac{4fL}{D}$, the pressure ratio, and the characteristics of Fanno flow [29]. Knowledge of Fanno flow is required for expressing the second term in equation (9.1).

The mass flow rate can be written as

$$\dot{m}_{out} = P_0(0) A M_{max} \frac{M_{in}(t)}{M_{max}} \left(\frac{P_0(0)}{P_0(t)}\right)^{\frac{k+1}{2k}} \sqrt{\frac{k}{RT_0(0)}} f[M_{in}(t)] \tag{9.5}$$

where

$$f[M_{in}(t)] = \left[1 + \frac{k-1}{2}(M_{in}(t))^2\right]^{\frac{-(k+1)}{2(k-1)}} \tag{9.6}$$

The Mach number at the entrance to the conduit, $M_{in}(t)$, is calculated by Fanno flow characteristics for the venting system resistance, $\frac{4fL}{D}$, and the pressure ratio. M_{max} is

[4] Fanno flow has been studied extensively, and numerous books describing this flow can be found. Nevertheless, a brief summary on Fanno flow is provided in Appendix A.

the maximum value of $M_{in}(t)$. In vacuum venting, the entrance Mach number, $M_{in}(t)$, is constant and equal to M_{max}.

Substituting equations (9.4) and (9.5) into equation (9.1), and rearranging, yields:

$$\frac{d\bar{P}}{d\bar{t}} = \frac{k\left(1 - \frac{t_{max}}{t_c}\bar{M}f(M_{in})\bar{P}^{\frac{k-1}{2k}}\right)}{1-\bar{t}}\bar{P}; \qquad \bar{P}(0) = 1. \tag{9.7}$$

The solution to equation (9.7) can be obtained by numerical integration for $\bar{P}$. The residual mass fraction in the cavity as a function of time is then determined using the "ideal gas" assumption. It is important to point out the significance of the $\frac{t_{max}}{t_c}$. This parameter represents the ratio between the filling time and the evacuation time. t_c is the time which would be required to evacuate the cylinder for a constant mass flow rate at the maximum Mach number when the gas temperature and pressure remain at their initial values, under the condition that the flow is choked, (The pressure difference between the mold cavity and the outside end of the conduit is large enough to create a choked flow.) and expressed by

$$t_c = \frac{m(0)}{AM_{max}P_0(0)\sqrt{\frac{k}{RT_0(0)}}} \tag{9.8}$$

Critical condition occurs when $t_c = t_{max}$. In vacuum venting, the volume pushed by the piston is equal to the flow rate, and ensures that the pressure in the cavity does not increase (above the atmospheric pressure). In air venting, the critical condition ensures that the flow is not choked. For this reason, the critical area A_c is defined as the area that makes the time ratio t_{max}/t_c equal to one. This can be done by looking at equation (9.8), in which the value of t_c can be varied until it is equal to t_{max} and so the critical area is

$$A_c = \frac{m(0)}{t_{max}M_{max}P_0(0)\sqrt{\frac{k}{RT_0(0)}}} \tag{9.9}$$

Substituting equation (9.2) into equation (9.9), and using the fact that the sound velocity can be expressed as $c = \sqrt{kRT}$, yields:

$$A_c = \frac{V(0)}{ct_{max}M_{max}} \tag{9.10}$$

where c is the speed of sound at the initial conditions inside the cylinder (ambient conditions). The t_{max} should be expressed by Eckert/Bar–Meir equation.

9.5 Results and Discussion

The results of a numerical evaluation of the equations in the proceeding section are presented in Figure **??**, which exhibits the final pressure when 90% of the stroke has elapsed as a function of $\frac{A}{A_c}$.

Parameters influencing the process are the area ratio, $\frac{A}{A_c}$, and the friction parameter, $\frac{4fL}{D}$. From other detailed calculations [4] it was found that the influence of the parameter $\frac{4fL}{D}$ on the pressure development in the cylinder is quite small. The influence is small on the residual air mass in the cylinder, but larger on the Mach number, M_{exit}. The effects of the area ratio, $\frac{A}{A_c}$, are studied here since it is the dominant parameter.

Note that t_c in air venting is slightly different from that in vacuum venting [3] by a factor of $f(M_{max})$. This factor has significance for small $\frac{4fL}{D}$ and small $\frac{A}{A_c}$ when the Mach number is large, as was shown in other detailed calculations [4]. The definition chosen here is based on the fact that for a small Mach number the factor $f(M_{max})$ can be ignored. In the majority of the cases M_{max} is small.

For values of the area ratio greater than 1.2, $\frac{A}{A_c} > 1.2$, the pressure increases the volume flow rate of the air until a quasi steady–state is reached. In air venting, this quasi steady–state is achieved when the volumetric air flow rate out is equal to the volume pushed by the piston. The pressure and the mass flow rate are maintained constant after this state is reached. The pressure in this quasi steady–state is a function of $\frac{A}{A_c}$. For small values of $\frac{A}{A_c}$ there is no steady–state stage. When $\frac{A}{A_c}$ is greater than one the pressure is concave upwards, and when $\frac{A}{A_c}$ is less than one the pressure is concave downwards. These results are in direct contrast to previous molds by Sachs , Draper , Veinik and Lindsey and Wallace , where models assumed that the pressure and mass flow rate remain constant and are attained instantaneously for air venting.

To refer to the stroke completion (100% of the stroke) is meaningless since 1) no gas mass is left in the cylinder, thus no pressure can be measured, and 2) the vent can be blocked partially or totally at the end of the stroke. Thus, the "completion" (end of the process) of the filling process is described when 90% of the stroke is elapsed. Figure **??** presents the final pressure ratio as a function $\frac{A}{A_c}$ for $\frac{4fL}{D} = 5$. The final pressure (really the pressure ratio) depends strongly on $\frac{A}{A_c}$ as described in Figure **??**. The pressure in the die cavity increases by about 85% of its initial value when $\frac{A}{A_c} = 1$ for air venting. The pressure remains almost constant after $\frac{A}{A_c}$ reaches the value of 1.2. This implies that the vent area is sufficiently large when $\frac{A}{A_c} = 1.2$ for air venting and when $\frac{A}{A_c} = 1$ for vacuum venting. Similar results can be observed when the residual mass fraction is plotted.

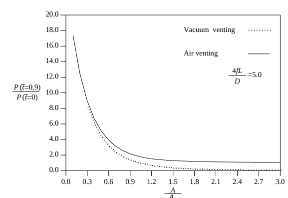


Fig. -9.3. The pressure ratios for air and vacuum venting at 90% of the piston stroke.

This discussion and these results are perfectly correct in a case where all the assumptions are satisfied. However, the real world is different and the assumptions have to examined and some of them are:

1. Assumption 1 is not a restriction to the model, but rather guide in the design. The engineer has to ensure that the resistance in the mold to air flow (and metal flow) has to be as small as possible. This guide dictates that engineer designs the

path for air (and the liquid metal) as as short as possible.

2. Assumptions 3, 4, and 10 are very realistic assumptions. For example, the error in using assumption 4 is less than 0.5%.

3. This model is an indication when assumption 5 is good. In the initial stages (of the filling process) the pressure is very small and in this case the pressure (force) to open the plates is small, and therefore the gap is almost zero. As the filling process progresses, the pressure increases, and therefore the gap is increased. A significant gap requires very significant pressure which occurs only at the final stages of the filling process and only when the area ratio is small, $\frac{A}{A_c} < 1$. Thus, this assumption is very reasonable.

4. Assumption 6 is associated with assumption 9, but is more sensitive. The change in the resistance (a change in assumption in 9 creates consequently a change in the plunger velocity. The plunger reaches the constant velocity very fast, however, this velocity decrease during the duration of the filling process. The change again depends on the resistance in the mold. This can be used as a guide by the engineer and enhances the importance of creating a path with a minimum resistance to the flow.

5. Another guide for the venting system design (in vacuum venting) is assumption 7. The engineer has to reduce the vent volume so that less gas has to be evacuated. This restriction has to be design carefully keeping in mind that the resistance also has to be minimized (some what opposite restriction). In air venting, when this assumption is not valid, a different model describes the situation. However, not fulfilling the assumption can improve the casting because larger portion of the liquid metal which undergoes mixing with the air is exhausted to outside the mold.

6. Assumption 8 is one of the bad assumptions in this model. In many cases there is more than one vent, and the entrance Mach number for different vents could be a different value. Thus, the suggested method of conversion is not valid, and therefore the value of the critical area is not exact. A better, more complicated model is required. This assumption cannot be used as a guide for the design since as better venting can be achieved (and thus enhancing the quality) without ensuring the same Mach number.

7. Assumption 9 is a partially appropriate assumption. The resistance in venting system is a function of Re and Mach numbers. Yet, here the resistance, $\frac{4fL}{D}$, is calculated based on the assumption that the Mach number is a constant and equal to M_{max}. The error due to this assumption is large in the initial stages where Re and Mach numbers are small. As the filling progress progresses, this error is reduced. In vacuum venting the Mach number reaches the maximum instantly and therefore this assumption is exact. The entrance Mach number is very small (the flow is even not choke flow) in air venting when the area ratio, $\frac{A}{A_c} >> 1$ is very large and therefore the assumption is poor. However, regardless

the accuracy of the model, the design achieves its aim and the trends of this model are not affected by this error. Moreover, this model can be improved by taking into consideration the change of the resistance.

8. The change of the vent area does affect the resistance. However, a detailed calculation can show that as long as the vent area is above half of the typical cross section, the error is minimal. If the vent area turns out to be below half of the typical vent cross section a improvement is needed.

9.6 Summary

This analysis (even with the errors) indicates there is a critical vent area below which the ventilation is poor and above which the resistance to air flow is minimal. This critical area depends on the geometry and the filling time. The critical area also provides a mean to "combine" the actual vent area with the vent resistance for numerical simulations of the cavity filling, taking into account the compressibility of the gas flow. Importance of the design also was shown.

9.7 Questions

Under construction

CHAPTER 10

Density change effects

In this appendix we will derive the boundary condition for phase change with a significant density change. Traditionally in die casting the density change is assumed to be insignificant in die casting. The author is not aware of any model in die casting that take this phenomenon into account. In materials like steel and water the density change isn't large enough or it does not play furthermore important role. While in die casting the density change play a significant role because a large difference in values for example aluminum is over 10%. Furthermore, the creation of shrinkage porosity is a direct consequence of the density change.

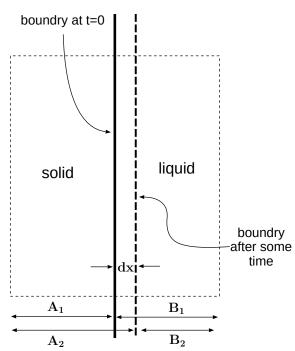


Fig. -10.1. The control volume of the phase change.

A constant control volume[1] is constructed as shown in figure 10.1. Solid phase is on the right side and liquid phase is on the left side. After a small time increment the moved into the the dashed line at a distance dx. The energy conservation of the control volume reads

$$\frac{d}{dt}\int_V \rho h\, dV = -\int_A \rho h v_i\, dA + \int_A k\frac{\partial T}{\partial n}\, dA \tag{10.1}$$

[1]A discussion on the mathematical aspects are left out. If explanation on this point will be asked by readers I will added it.

Analogy the mass conservation for the control volume is

$$\frac{d}{dt}\int_V \rho\, dV = -\int_A \rho v_i\, dA \tag{10.2}$$

The equations (10.1) and (10.2) do not have any restrictions of the liquid movement which has to be solved separately. Multiply equation (10.1) by a constant h_l results in

$$\frac{d}{dt}\int_V \rho h_l\, dV = -\int_A \rho h_l\; v_i\, dA \tag{10.3}$$

Subtraction equation (10.3) from equation (10.1) yields

$$\frac{d}{dt}\int_V \rho (h - h_l)\, dV = -\int_A \rho (h - h_l) v_i\, dA + \int_A k \frac{\partial T}{\partial n}\, dA \tag{10.4}$$

The first term on the right hand side composed from two contributions: one) from the liquid side and two) from solid side. At the solid side the contribution is vanished because $\rho(h-h_l)v_i$ is zero due to v_i is identically zero (no movement of the solid, it is a good assumption). In the liquid phase the term $h - h_l$ is zero (why?) thus the whole term is vanished we can write the identity

put explanation or question

$$\int_A \rho (h - h_l) v_i\, dA \equiv 0 \tag{10.5}$$

where v_i is the velocity at the interface.

maybe the derivations are too long. shorten them?

The first term of equation (10.4) can be expressed in the term of the c.v.[2] as

$$\begin{aligned}\frac{d}{dt}\int_V \rho (h - h_l)\, dV &= \frac{\overbrace{\rho_s A_2 (h_s - h_l) - \rho_s A_1 (h_s - h_l)}^{solid} + \overbrace{(\cdots (h_l - h_l))}^{liquid=0}}{dt} \\ &= (\rho_s (h_s - h_l))\frac{dx}{dt} = \rho_s (h_s - h_l) v_n \end{aligned} \tag{10.6}$$

liquid side contribution is zero since $h - h_l \equiv 0$ and the solid contribution appears only in transitional layer due to transformation liquid to solid. The second term on right hand side of equation (10.4) is simply

$$\int_A k \frac{\partial T}{\partial n}\, dA = k_s \frac{\partial T}{\partial n} - k_l \frac{\partial T}{\partial n} \tag{10.7}$$

Thus, equation (10.4) is transformed into

$$\rho_s (h_s - h_l) v_n = k_s \frac{\partial T}{\partial n} - k_l \frac{\partial T}{\partial n} \tag{10.8}$$

It is noteworthy that the front propagation is about 10previously was calculated. Equation (10.7) holds as long as the transition into solid is abrupt (sharp transition).

[2]please note some dimensions will canceled each other out and not enter into equationssss

Meta

For the case of where the transition to solid occurs over temperature range we have create three zones. Mathematically, it is convenient to describe the the mushy zone boundaries by two boundary conditions.

Meta End

Meta

The creation of voids is results of density changes which change the heat transfer mechanism from conduction to radiation. The location of the void depends on the crystallization and surface tension effect, etc. The possibility of the "liquid channels" and the flow of semi-solid and even solid compensate for this void.

Meta End

Klein's paper

Meta

Yet, one has to take into consideration the pressure effect The liquidation temperature and the latent heat are affected somewhat by the pressure. At pressure between the atmospheric to typical intensification pressure the temperature and latent heat are effected very mildly. However, for pressure near vacuum the latent heat and the temperature are effected more noticeably.[3]

Meta End

The velocity of the liquid metal due to the phase change can be related to the front propagation utilizing the equation (10.2). The left hand side can be shown to be $(\rho_s - \rho_l)v_n$. The right hand side is reduced into only liquid flow and easily can be shown to be $\rho_l v_l$.

$$
\begin{aligned}
(\rho_s - \rho_l)\, v_n + \rho_l v_l &= 0 \\
(\hat{\rho} - 1) &= \frac{v_l}{v_n}
\end{aligned}
\tag{10.9}
$$

where $\hat{\rho}$ is the density ratio, ρ_s/ρ_l.

[3] I have used Clapyron's equation to estimate the change in temperature to be over 10 degrees (actually about 40^0[C]). However, I am not sure of this calculations and I had not enough time to check it in the literature. If you have any knowledge and want to save me a search in the library, please drop me a line.

CHAPTER 11

Clamping Force Calculations

Under construction

It doesn't matter on what machine the product is produce, the price is the same

Prof. Al Miller, Ohio

CHAPTER 12

Analysis of Die Casting Economy

12.1 Introduction

The underlying reason for the existence of the die casting process is so that people can make money. People will switch to more efficient methods/processes regardless of any claims die casting engineers make[1]. To remain competitive, the die casting engineer must totally abandon the "Detroit attitude," from which the automotive industry suffered and barely survived during the 70s. The die casting industry cannot afford such a luxury. This topic is emphasized and dwelt upon herein because the die casting engineer cannot remain stagnate, but rather must move forward. It is a hope that the saying *"We are making a lot of money— why should we change?"* will totally disappear from the die casting engineer's jargon. As in the dairy industry, where keeping track of specifics created the "super cow," keeping track of all the important information plus using scientific principles will create the "super die casting economy." This would be true even if a company, for marketing reasons, needed to offer a wide variety of services to their customers. Which costs the engineer can alter, and what he/she can do to increase profits, are the focus of this chapter.

First as usual a discussion on the "common" model is presented, the validity and the usefulness is discussed, and finally a proper model is unveiled.

12.2 The "common" model, Miller's approach

They started with idea that the price is effected by the following parameters: 1)weight, 2)alloy cost, 3)complexity, 4)tolerance, 5)surface roughness, 7)aspect ratio, 8)produc-

[1]The DDC, a sub set of NADCA operations, is now trying to convince die casting companies to advertise through them to potential customers. Is the role of the DCC or NADCA to be come the middle man? I do not think so. The role of these organizations should be to promote the die casting industry and not any particular company/ies.

tion quantity, and 9)"secondary" machining. After statistical analysis they have done they come–out with the following equation

$$\begin{aligned} price = 0.485 + 2.20weight - 0.505zinc + 0.791mag + 0.292details \\ +0.637tolerance - 0.253quantity \end{aligned} \tag{12.1}$$

where mag, zinc, details(<100 dimension), and tolerance are on/off switch. They claim that this formula is good for up to ten pounds (about 4.5[kg]). In summary, if you expect to get equation that does not have much with the actual cost, you got one.

12.3 The validity of Miller's price model

There is a saying garbage in garbage out. The proponent conclusion from equation 12.2 is that it does not matter how good the design how much scrap the product generates the price is the same. This is exactly what we are preaching against. The question must be asked, how they calculate the average price of the product that statistically they analyzed, if they have no idea how to the calculate the actual price in the first place. So, how they determine that the product will produce profit if the price have no relationship to the actual production cost?

The "critical/optimum point" is the point above which the quality is good and below which the quality is unacceptable. As it turns out, much above and just above the critical point produces an acceptable quality product for many design parameters in the die casting process. However, the cost is considerably higher[2]. The hydraulic diameter of the runner system is one such example (see Figure **??**). The price of the runner system (scrap) is proportional to the hydraulic diameter squared, $\propto {H_D}^2$ (a parabola), as shown by the "scrap cost" curve in Figure **??**. The machine cost is constant (as a first approximation) up to the point below which the machine cannot produce an acceptable quality. The engineer would like to design the runner diameter just above this point. "Machine" cost as a function of the runner diameter for several different machines is shown by the "machine" curves in Figure **??**. The combined cost of the scrap and the machine usage can be drawn, and clearly the combined–cost curve has a minimum point, and is referred to here as the "optimum" point[3]. This is a typical example of how a design parameter (runner hydraulic diameter) effects the cost and quality.

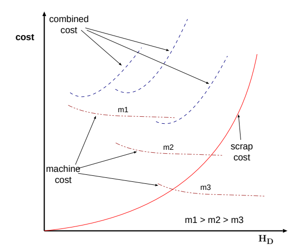


Fig. -12.1. Production cost as a function of the runner hydraulic diameter.

The components of the production cost now should be dissected and analyzed, and then a model will be constructed. It has to be realized that there are two kinds of

[2]The price of a die casting machine increase almost exponentially with the machine size. Thus, finding the smallest die casting machine to run the job is critical importance.

[3]The change in the parts numbers per shot will be discussed in section **??**.

cost components: 1) those which the engineer controls, and 2) those which the engineer does not control. The uncontrolled components include overhead, secondary operations, marketing, space[4], etc. This category should be considered as a constant, since the engineer's actions/choices do not affect the cost and therefore do not affect the cost of design decisions. However, the costs of die casting machine capital and operations, personnel cost, melting cost, and scrap cost[5] are factors which have to be considered, and are discussed in the succeeding sections. In this analysis it is assumed that the die casting company is here to make a buck, and it is also assumed that competitive price wars for a specific project and/or any other personal reasons influencing decision making are not relevant[6].

This issue is formulated in such a way that the engineer will have the needed tools to make appropriate decisions.

12.4 The combined Cost of the Controlled Components

The engineer has to choose the least expensive machines available, yet produce a product of acceptable quality. The least expensive machine has to chosen. The price for production cost each machine is determined from the sums of every component. If the customer is in a rush, the cost should be calculated for the available die casting machine as follows:

$$\omega_{total} = \sum_i \omega_i \tag{12.2}$$

12.5 Die Casting Machine Capital Costs

The capital cost of a die casting machine (like any other industrial equipment) has two components: 1) money cost and 2) depreciation cost. The money cost in many cases is also comprised of two components: 1) loan cost and 2) desired profit[7]. The cost of a loan is interest. The value of the interest rate is easy to evaluate – just ask a banker. However, the value of the desired profit is harder to estimate. One possible way to estimate this is by checking how much it costs to lease a similar machine. Adding these two numbers yields a good estimate of the money costs. In today's values, the money cost value is about 12%–25%. Depreciation is a loss in value of the die casting machine[8].

[4]The room–amount cost for the machine is almost insensitive to the engineer's choice of the size or brand of the die casting machine

[5]See the discussion on this topic in section 12.7 page 137 for the more detail.

[6]such as doing a project to keep a customer for another project are not relevant here. Yet, this information can be used to make intelligent decision in regard to the customer.

[7]This profit is different from the operational profit. For example, if one own a taxi, he should have two kind of profits: 1) those from owning the taxi and 2) those from operating the taxi. He can rent the taxi and have a profit just for owning the vehicle. The owner should earn additional income for the eight hours shift. These refereed herein as operational earnings.

[8]The effects of taxes on the depreciation analysis are sometime significant, but to reduce the complexity of the explanation here, it is ignored.

In this analysis, it is assumed (or at least hoped) that the other die casting machines have other jobs waiting for them. If a company for a short time is not working to full capacity, the analysis will still be valid with minor modifications. However, a longer duration of being below full capacity requires the company to make surgical solutions.

The cost of the die casting machine depends on the market and not on the value the accountant has put on the books for that machine. Clearly, if the machine is to be sold/leased, the value obtained will be according to the market as "average" value. The market value should be used since the machine can be sold and this money can be invested in other possibilities. Amortization is estimated in the same manner. The difference between the current value and the value at one year older is the depreciation value.[9] Having these numbers, the capital cost can be estimated. For example, a one million dollar machine with a 20% money cost and a 5% depreciation cost equals about $250,000.00 a year. To convert this number to an hourly base rate, the number of idle days (on that specific machine) is required, and in many case is about 60 – 65 days. Thus, hourly capital cost of that specific machine is about $34.70.

A change in the capital cost per unit can be via the change in the cycle time. The change in the cycle time is determined mostly by the solidification processes, which are controlled slightly by the runner design. Yet this effect can be diminished by controlling the cooling rate. Hence, the capital price is virtually unaffected once the die casting machine has been selected for a specific project. Here, the cost per unit can be expressed as follows:

$$\omega_{capital} = \frac{\text{capital cost per hour}}{N_c N_p} \tag{12.3}$$

where $\omega_{capital}$ is the capital cost per unit produced, N_c is the number of cycles per hour, and N_p the number of parts shot.

12.6 Operational Cost of the Die Casting Machine

Operational costs are divided into two main categories: 1) energy cost, and 2) maintenance cost. The energy cost is almost insensitive to the mold/runner design. The maintenance cost is determined mostly by the amount of time the die casting machine is in operation. This cost is comprised of the personnel cost of doing the work, hydraulic fluid maintenance, components (ladle, etc.) and maintenance, etc., which is different for each machine and company. However, the value of this cost can be considered invariant for a specific machine in regard to design parameters. The engineer's duty is to calculate the operation cost for every die casting machine that is in the company. This can be achieved by keeping records of the maintenance for each machine and adding up all related costs performed on that machine in the last year.

The energy costs are the costs of moving the die casting machine and its parts and accessories. The energy needed to move all parts is the electrical energy which can easily be measured. Today, electrical energy costs are far below one dollar for one [kW]×hour (0.06-0.07 of a dollar according to NSP prices). Even a large job will require

[9] This also depends on changes in the condition of the machine.

less than 10[kW×hour]. Thus, the total energy cost is in most cases at most $1.00 per hour. The change in the energy is insensitive to the runner and venting system designs and can vary by only 30% (15 cents for a very very large job), which is insignificant compared to all other components. The operation cost can be expressed as

$$\omega_{operation} = \frac{\text{operation cost per hour f(machine size, type etc.)}}{N_c N_p} \tag{12.4}$$

12.7 Runner Cost (Scrap Cost)

The main purpose of the runner is to deliver the liquid metal from the shot sleeve to the mold, since the mold cannot be put (hooked) directly on (to) the shot sleeve. The requirements of the runner have conflicting demands. Here is a partial list of the requirements for the runner:

1. As small as possible so it will create less scrap.
2. Large enough so that there is less resistance in the runner to the liquid metal flow, so that the job can be performed on a smaller die casting machine.
3. Small enough so that the plunger will need to propel only a minimum amount of liquid metal. In a way this is the same as requirement 1 above but less important.

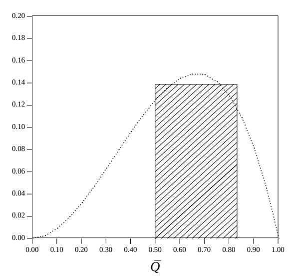


Fig. -12.2. The reduced power of the die casting machine as a function of the normalized flow rate.

Clearly, a large runner volume creates more scrap and is a linear function of the size of the runner volume, which is

$$V_{runner} = \overbrace{\frac{\pi {H_{DT}}^2}{4}}^{area} \overbrace{L_T}^{length} \tag{12.5}$$

where H_D is the typical size of the hydraulic diameter, and L_T its length (these values are not the actual values, but they are used to represent the sizes of the runner). From equation (12.5), it is clear that the diameter has one of the greater impacts on the scrap cost. The minimum diameter at which a specific machine can produce good quality depends on the required filling time, gate velocity, other runner design characteristics, and the characteristics of the specific machine.

Scrap cost is a linear function of the volume[10]. The scrap cost per volume/weight consists of three components: 1) the melting cost, 2) the difference between the buying price and the selling price (assuming that the scrap can be sold), and 3) the handling

[10] Up to a point about which it becomes more sensitive to the volume.

cost. The melting cost includes the cost to raise the metal temperature to the melting point, to melt the metal, and to hold the metal temperature above the melting point. The melting cost can be calculated by measuring energy used (crude oil or natural oil in most cases) plus the maintenance cost of the furnace divided by the amount of metal that has been casted (the parts and design scrap). The buying price is the price paid for the raw material; the selling price is the price for selling the scrap. Sometime it is possible to reuse the scrap and to re–melt the metal. In some instances, the results of reusing the scrap will be a lower grade of metal in the end product. If reuse is possible, the difference in cost should be substituted by the lost metal cost, which is the cost of 1) metal that cannot be recycled and 2) metal lost due to the chemical reactions in the furnace. The handling cost is the cost encountered in selling the metal, and it includes changing the mechanical or chemical properties of the scrap, transportation, cost of personnel, storage, etc. Each handling of the metal costs a different amount, and the specifics can be recorded for the specific metal.

Every job/mold has typical ranges for the filling time and gate velocity. Moreover, a rough design for the runner system can be produced for the mold. With these pieces of information in place, one can calculate the gate area (see pQ^2 diagram calculations in Chapter 7 for more details, and this part is repeated in that Chapter. I am looking for the readers input to decided what is the best presentation.). Then the flow rate for the mold can be calculated by

$$Q = \frac{V_{mold}}{A_{gate} V_{gate}} \tag{12.6}$$

Additionally, the known design of the runner with flow rate yields the pressure difference in the runner, and this yields the power required for the runner system,

$$\mathcal{P}_r = Q \Delta P \tag{12.7}$$

or in normalized form,

$$\overline{\mathcal{P}_r} = \frac{\overline{Q} \Delta P}{P_{max} \times Q_{max}} \simeq \overline{Q} \times \overline{P} \tag{12.8}$$

Every die casting machine has a characteristic curve on the pQ^2 diagram as well. Assuming that the die casting machine has the "common" characteristic, $\overline{P} = 1 - \overline{Q}^2$, the normalized power can be expressed

$$\overline{\mathcal{P}_m} = \overline{Q}(1 - \overline{Q}^2) = \overline{Q}^2 - \overline{Q}^3 \tag{12.9}$$

where $\overline{\mathcal{P}_m}$ is the machine power normalized by $P_{max} \times Q_{max}$. The maximum power of this kind of machine is at 2/3 of the normalized flow rate, $\overline{Q}$, as shown in Figure **??**. It is recommended to design the process so the flow rate occurs at the vicinity of the maximum of the power. For a range of $1/3$ of $\overline{Q}$ that is from $0.5\overline{Q}$ to $0.83\overline{Q}$, the average power is 0.1388 $P_{max}Q_{max}$, as shown in Figure **??** by the shadowed rectangular. One may notice that this value is above the capability of the die casting machine in two ranges of the flow rate. The reason that this number is used is because with some improvements of the the runner design the job can be performed on this machine, and there is no need to move the job to a larger machine.

If the machine power turns out to be larger than the required power of the runner, $\overline{\mathcal{P}_m} > C_s\overline{\mathcal{P}_r}$, the job can then be performed on the machine; otherwise, a bigger die casting machine is required. In general, the number of molds castable in a single cycle is given by

$$N_p = \left\lfloor \frac{\mathcal{P}_m}{C_s \mathcal{P}_r} \right\rfloor = \left\lfloor \frac{\overline{\mathcal{P}_m}}{C_s \overline{\mathcal{P}_r}} \right\rfloor \tag{12.10}$$

The floor symbol "⌊" being used means that the number is to be rounded down to the nearest integer. C_s denotes the safety factor coefficient. In the case that N_p is less than one, $N_p > 1$, that specific machine is too small for this specific job. After the number of the parts has been determined (first approximation) the runner system has to be redesigned so that the required power needed by the runner can be calculated more precisely. Plugging the new numbers into equation (12.10) yields a better estimation of the number of parts. If the number does not change, this is the number of parts that can be produced; otherwise, the procedure must be repeated.

In this analysis, the required clamping forces that the die casting machine can produced are not taken into consideration. Analysis of the clamping forces determines the number of possible parts and it is a different criterion which required to satisfied, this will be discussed in more detail in Chapter 11. The actual number of parts that has to be taken into consideration is the smaller of the two criteria. Next, the new volume of the runner system has to be calculated. The cost per cavity is the new volume divided by the number of cavities:

$$\omega_{scrap} = \frac{V_{runner} \times (\text{cost per volume})}{N_p} \tag{12.11}$$

12.8 Start–up and Mold Manufacturing Cost

The cost of manufacturing of a mold is affected slightly by the the shape of the runner. The only exemption to the above statement is the effect of change of the cross section shape and size on the cost of manufacturing which will be discussed in Chapter 6. A larger part of expense is the start–up time cost which is composed of 1) rebuilding the mold, 2) lost time (personnel time, machine time, etc), and 3) lost material. When dealing with calculation of the start–up time two things have to be taken into account 1) the ratio of the start up cost to the total cost, and 2) how long it is expected to take to achieve a product of acceptable quality. The cap cost has to be determined from the total cost per unit, and then multiplied by the total number of units. This number is the net production cost. The start–up cost cannot (should not) exceed 10%–15% of that number. Presently, it is very hard to determined the number of trials that will required per mold. This number is related to the complexity of the shape. The more complex the shape is, the more likely it is that the number of attempted "shots" will increase. If it is assumed that the engineer is experienced, the only factor that will affect the number of shots will be the complexity – provided that the job can be performed on the same die casting machine. The complexity of the shape should present a general idea of the number of expected attempts, and should be used in calculating the start–up cost,

$$\omega_{startUp} = \frac{\text{(Cost per attempt)} \times N_a}{N_r} \tag{12.12}$$

where N_a is the number of attempts, and N_r is the number of the total parts to be produced.

12.9 Personnel Cost

The cost of personnel is affected by the cycle time plus the number of parts produced per cycle. With today's automatization, the number of operators is decreasing. In some companies, one operator controls three or more machines. Hence, the personnel cost is:

$$\omega_{personnel} = \frac{\text{salary per hour}}{\text{number of machines} \times \text{number of cycle}} \tag{12.13}$$

In toady's market, the operator cost is in the range of \$10–\$20 per hour. When automatization is used, the personnel cost is significantly reduced to the point that it is insignificant.

12.10 Uncontrolled components

The price to be charged to the customer has to include the uncontrolled components as well. There are several methods for adding this fragment to the part cost. First, the total cost of the uncontrolled components has to be calculated. This can be done by adding up the costs from the previous year and estimated for this year This cost includes salaries that were paid in the last year plus the legal expenses, rent, and marketing, etc. Dividing the uncontrolled components of cost has many reasonable options. Here is a selected list according to:

- the number of parts
- the number of parts and their size/weight
- the number of the parts and their complexity

12.11 Minimizing Cost of Single Operation

In this section several issues related to cost minimization and/or profit increases are discussed. An example of such problem is when an engineer has to make a decision about supplies. Ordering a supply commonly to has to do with two or more conflicting costs. These two conflicting costs have a minimum of ordering cost associated with an optimum number of orders. Consider a simple situation where ABC company produces x devices per year, for example x hard drive frames per year. These x devices require y items supporting components. For example, these hard drive frames are made of y mass of aluminum[11]. So, the ABC company has to order y items supporting components a year. It is assumed that the number of items ordered and consumed per year is constant[12]

In a typical situation, these items are ordered several times, the cost is composed of two components. The first component is the cost per order which includes such things as the delivery cost, the time to order (verification if it is in stock). The second component is associated with the cost of keeping the stock beyond one day supply. This category includes such as the money tied to material, the storage used (cost of space and handling) etc. For simplicity, it is assumed that the cost per delivery is constant and does not change during the year. It also assumed that the storage per day cost is also constant. To illustrate this point, consider two extreme cases of ordering. One possibility is to order everyday and other possibility is to order once per year. The cost for ordering very day is 365[13] times the ordering cost. In this case, there is no stocking cost. The other extreme is one order a year for which there is one time order cost with 364 days of stocking cost.

Example 12.1:

Under conditions below, calculate the ordering cost for ordering every day of the year and once a year. The delivery cost of aluminum is $150 per delivery. The cost of storing a kilogram aluminum is 0.10 dollars. And the cost of a kilogram aluminum is 10 dollars. The daily aluminum consumption is 750 kilogram. The money cost is 0.01 percent a day.

<u>SOLUTION</u>

The ordering cost is $150. Hence, the cost for the first case is composed of the ordering cost (in this case only delivery cost) only. The stocking cost is vanished because no stocking is involved. Thus, the total cost is

$$\text{total cost} = 365 \times 150 = \$54,750 \tag{12.I.a}$$

The ordering occurs once a year with 364 days of stocking cost which must be added.

[11] Or the weight for the total of the hard drives [in kilograms].

[12] If the number is not constant or even seasonal (depend on the season) this model can be expanded by numerical analysis.

[13] It is assumed that the year is 365 days which about 75% of the occurrence. There are years with 356 days.

The daily stocking cost one day consumption is

$$\begin{array}{c}\text{stocking cost}\\\text{per day}\end{array} = \overbrace{750}^{\substack{\text{stocked}\\\text{material}\\\text{amount}}} \times \overbrace{10.0}^{\substack{\text{cost per}\\\text{kilogram}}} \times 0.0001 = \$0.75 \tag{12.I.b}$$

On the first day has 364 portions (days) to be stocked. On the second day has 363 portions (days) to be stocked. On every sequence day there is one less portions to be stocked. Hence, there is a series of 364 items which starts with 364 and end at zero. This exactly algebraic series which can be calculated as

$$\begin{array}{l}\text{total}\\\text{portions}\end{array} = \frac{a_1 + a_{364}}{2}\,364 = \frac{364+0}{2} * 364 = 66248 \tag{12.I.c}$$

Thus, the total cost for ordering once is

$$\begin{array}{c}\text{total cost}\\\text{for once}\\\text{ordering}\end{array} = \$150 + 66248 \times 0.75 = \$49836 \tag{12.I.d}$$

For the purpose of this example, the choice of one time order is better. While the cost of one time ordering looks better in this example, in real life other factors should be considered. In this case case, the erosion of the company credit which was not a considered here. That fact should reversed the decision.

End Solution

Example 12.1 exhibits the effect of the number of orders on the cost of operation. Intuitively, for example, it can be observed that for two (2) orders, as compare to one order, the cost is reduced by half plus \$150 (why?).

The focus of this discussion is to find the optimum number of times to order the items per year. Not all years are the same (some years are 366 days) but here it will be assumed that all years are 365 days. In this discussion, months and weeks do not appear and the dissimilarity is not discussed. The number of items ordered per year is assumed to be constant for this discussion and is denoted as N. Hence, the number of order items per day is $N/365$. The cost per day of store of item is denoted y. The ordering cost is r. The unknown number of periods is denoted as p.

The number of days per period is

$$D = \frac{365}{p} \tag{12.14}$$

The period cost of storage is the number parts not used which were kept for the following days of the period. The number of items remained for the storage for the first day of the period are

$$Remainder = (D-1)\cdot\frac{N}{365} \tag{12.15}$$

On the second day, the number of items that remain in storage is

$$Remainder = (D-2)\,\frac{N}{365} \tag{12.16}$$

There are no items to stored on the last day of the period. The number of days that we had to store is $(D-1)$ (no need to store at the last day). The cost of the storage per period is

$$\begin{array}{c}\text{number of}\\ \text{items}\end{array} = \overbrace{(D-1)\,\frac{N}{365}}^{\text{first day}} + \overbrace{(D-2)\,\frac{N}{365}}^{\text{second day}} + \cdots + \overbrace{2\,\frac{N}{365}}^{\text{3rd last day}} + \overbrace{1\,\frac{N}{365}}^{\text{day before last}} + \overbrace{0\,\frac{N}{365}}^{\text{last day}} \tag{12.17}$$

Equation (12.17) can be rearranged (because it is a regular algebraic series) is

$$\begin{array}{c}\text{number of}\\ \text{items}\end{array} = \frac{N}{365}\sum_{i=1}^{D-1} D-1-i \tag{12.18}$$

or

$$\begin{array}{c}\text{number of}\\ \text{items}\end{array} = \overbrace{\frac{(D-1)+1}{2}}^{\text{averaged to be stored days}}\ \overbrace{(D-1)}^{\text{number of days}}\ \frac{N}{365} = \frac{D\,(D-1)}{2}\,\frac{N}{365} \tag{12.19}$$

The cost per period is

$$\begin{array}{l}\text{storage cost}\\ \text{per period}\end{array} = \frac{D\,(D-1)}{2}\,\frac{N}{365}\,y \tag{12.20}$$

Or in terms of the number of period, p (see also equation (12.14)) the storage cost is

$$\begin{array}{l}\text{storage cost}\\ \text{per period}\end{array} = \frac{\frac{365}{p}\left(\frac{365}{p}-1\right)}{2}\,\frac{N}{365}\,y \tag{12.21}$$

The yearly storage cost is

$$\left(\begin{array}{c}\text{yearly storage}\\ \text{cost}\end{array}\right) = \left(\begin{array}{c}\text{storage cost}\\ \text{per period}\end{array}\right)\times p = \frac{N}{2}\left(\frac{365}{p}-1\right)y \tag{12.22}$$

The total cost yearly ordering cost, C, is

$$C = \left(\begin{array}{c}\text{yearly}\\ \text{storage cost}\end{array}\right) + \left(\begin{array}{c}\text{yearly}\\ \text{ordering cost}\end{array}\right) \tag{12.23}$$

In term of the number of periods the total ordering cost is

$$\left(\text{ yearly cost }\right) = r\,p + \frac{N}{2}\left(\frac{365}{p} - 1\right) y \tag{12.24}$$

The minimum cost expressed by the expression (12.24) a derivative with respect to p number of period as

$$\frac{\partial C}{\partial p} = r + \frac{N\,y\,365}{2}\left(-\frac{1}{p^2}\right) = 0 \tag{12.25}$$

The solution of equation (12.25) is

$$p = \sqrt{\frac{N\,y\,365}{2\,r}} \tag{12.26}$$

These calculations were made under the assumption that the number of periods is a real number. However, the number of periods and several other parameters must be an integer. It can be argued that the number of orders can be 6.5 on the account that in two years planning to have 13 orders $(13/2 = 6.5)$ It is more common to have solution with a totally irrational number which leads in practicality solution that cannot be used. The real solution (in a yearly integer planning sense) lay either on one of adjoining sides of the continuous solution. It has to be manually calculated. The calculation can yield a number of periods to be below one. The actual meaning is that the ordering cost is significant (dominate) so the order must be continuous. On the other extreme, when the ordering cost is so insignificant to order can occur several time a day.

Example 12.2:
In ABC die casting company has 68000 kg of aluminum a year. The cost of storage of 1 kg a day is \$0.04 and cost of ordering is \$130. What is the optimum order period?

<u>Solution</u>

The information provides that

$$p = \sqrt{\frac{68000 \times 0.04 \times 365}{2 \times 130}} = 3.23443016 \tag{12.II.a}$$

The solution is either 3 or 4 times when additional consideration has to be taken into account.

End Solution

12.12 Introduction to Economics

The main goal of any company is to increase the total profits which is denoted as T. The total profits, T, is a function of (x) the number of the sold items. If the stock or inventory is assumed to be zero or at least constant, the cost (C) and the revenue (R) are a pure function of the number of items sold, x. The aim of this discussion to find the optimum number of items that will yield the maximum profits.

A new concept is introduced in this discussion, the marginal profits. The marginal profits refers to the point where additional production actually reduces the profit. This concept requires a new tool the discrete mathematics. In the previous section, this concept was used in section 12.11 without proper introduction. The discrete mathematics can be described as treading integer phenomena as a continuous while recognizing that it is integer. The practical application is the "virtual" maximum or minimum is found assuming continuous mathematics but later the actual maximum or minimum is found by looking the sides of the previous found virtual solution.

Thus, the profits can be written regardless whether the number of items is integer or continuous as

$$T(x) = R(x) - C(x) \tag{12.27}$$

The revenue is a function of items number, x, as well as the price, $P(x)$ is also a function of x. The typical cost, P, is composed from several components such as the fix cost, linearly associated with the number of items, and cost that associated with not linear with the number of items. The first cost is relatively obvious, while the differentiation between the second and third cost as to explain. The second cost is referred to the cost that occur for every item. For example, the aluminum consumed for every frame produced depends only the number of item produce (approximately). On the other hand, the cost production of the mold for the frame depends on the number of the items. Thus, the mold cost is inversely proportional $(1/x)$ to the number of items. The are elements of the cost that depends on the run size. These costs can be written as

$$C(x) = \overbrace{\mathcal{F}_c}^{\text{fixed}} + \overbrace{\mathcal{L}}^{\text{cost per item}} x + \overbrace{F(x)}^{\text{more complex situation}} \tag{12.28}$$

Hence, the cost per item ($\mathcal{L}$ is the linear cost per item and is not a function of x.) is

$$\overline{c}(x) = \frac{C(x)}{x} = \frac{\mathcal{F}_c}{x} + \mathcal{L} + \frac{F(x)}{x} \tag{12.29}$$

Examining the terms in equation (12.29) reveals that some terms vanish as the number of items increase to infinity $x \longrightarrow \infty$, $(1/x) = 1/\infty \sim 0$. Any term that behaves like $1/x^n$ when n is positive vanishes. Seldom there are terms that that do not vanish. A typical equation repressing this situation is

$$C(x) = 10,000 + 45\,x + 100\sqrt{x} \tag{12.30}$$

when the results are in dollars or other currency.

The revenue is a strong function of the items sold. The price id determine by the supply and demand diagram (origin of the pQ^2 diagram). The interesting part of the supply and demand diagram shows that the price decreases as the number of items increases. This part of the analysis indicates that the profits are a strong function item parts. These facts shows that the total profits is reduced when the total revenue is increased.

The supply and demand diagram was proposed by Marco Fanno (the older brother of Fanno from Fanno flow). Normally the supply and demand diagram is not determined at the factory rather is determined by the market forces or trends. Sometime this diagram is suggested by supplier[14]. Hence the revenue typically appears as

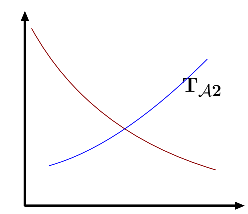


Fig. -12.3. Supply and Demand.

$$R(x) = p(x) \sim P_0 - P_r\, x \tag{12.31}$$

Where P_0 is initial price of the item small quantity, and a P_c is the reduction of the price which is a function of the sold items. P_r does not have to be a continues function.

12.12.1 Marginal Profits

One way to look at the suggested price, the alternative to supply and demand diagram price, is examining the marginal profits. Suppose that the company discussed here selling a certain number of devices x. Should the company increase the number of sales. This question leads indirectly to the suggested price (by the supplier/manufacture). The marginal profits is defined as

$$\frac{dT}{dx} = \lim_{x\to 0} \frac{\Delta T}{\Delta x} \tag{12.32}$$

It can be noticed that here the virtual continues assumption is invoked. Clearly, the minimum number, x of items is one. However, for large number of items one can be consider small enough. In other cases, when the number of item is not so small there is an error. However, the sequential operation helps to rectify it. In the same vein several other definitions can be defined. The marginal cost is defined as

$$\frac{dC}{dx} = \lim_{x\to 0} \frac{\Delta C}{\Delta x} \tag{12.33}$$

The marginal price is defined in the same manner.

Example 12.3:

The cost function was determined to be $C(x) = 10,000 + 3.5\,x + 50\,\sqrt[5]{x}$. Calculate the average cost and marginal cost for this given function at 2000 items.

[14]For example, when you buy item the supplier state a price for quantity between 1-5 items you pay full price but you buy between 6-10 items you have 5% discount per item. Buying between 10-20 items gives additional 5%.

SOLUTION

The average cost is determined by the dividing the total cost by the number of items

$$c(x) = \frac{C(x)}{x} = \frac{10,000 + 3.5\,x + 50\,\sqrt[5]{x}}{x} \tag{12.III.a}$$

The marginal cost is determined by equation (12.33) to be

$$\frac{dC}{dx} = 3.5 + \frac{50}{5}\,x^{-4/5} \tag{12.III.b}$$

At 2000 items the averaged cost is $\$8.61$ and the marginal cost is $\$3.61$. Notice the significance or the reduction in the profits.

End Solution

The price function is can be estimated from the feel for the market. This can be illustrated by the following example.

Example 12.4:

Your costumer suggest that if you reduce the price by 50¢ he will buy more 200 more in addition to 2000 that already ordered. The current price is $4. Determine the price function as a function of x, number of units sold, . What is the number weekly units that will maximize the weekly revenue? Calculate the maximum weekly revenue.

SOLUTION

This stage involves some extrapolations of the data which was provided. It is unknown but can be assumed that relationships are linear and can be extent to a larger range. In make this assumption, one does not calculate the exact value but rather predicts and trends in which great for uncertain situations. It was provided that the number of items

$$x = 2000 + \frac{4 - p(x)}{0.5} \times 200 \tag{12.IV.a}$$

Equation (12.IV.a) can be rearranged as

$$p(s) = 4 - \frac{x - 2000}{400} = 11 - 0.0025\,x \tag{12.IV.b}$$

The revenue can be expressed as

$$R(x) = p(x)\,x = 11\,x - 0.0025\,x^2 \tag{12.IV.c}$$

It can be noticed, not only the profit are reduced (since the difference between production cost and revenue) but the revue is reduce when the price is increased.

$$\frac{R(x)}{dx} = 11 - 0.005\,x \Longrightarrow= x_{max} = \frac{11}{0.005} = 1200 \tag{12.IV.d}$$

End Solution

12.13 Summary

In this chapter the economy of the design and choices of the casting process have been presented. It is advocated that the "averaged" approach commonly used in the die casting industry be abandoned. This chapter is planed to be the "flagship" chapter of this book. Engineer should adopt a more elaborate method, in which more precise calculations are made, to achieve maximum profit for there company. It is believed that the new method will create the "super die casting economy."

12.14 Question

APPENDIX A

Fanno Flow

An adiabatic flow with friction is named after Ginno Fanno a Jewish engineer. This model is the second pipe flow model described here. The main restriction for this model is that heat transfer is negligible and can be ignored [1]. This model is applicable to flow processes which are very fast compared to heat transfer mechanisms with small Eckert number.

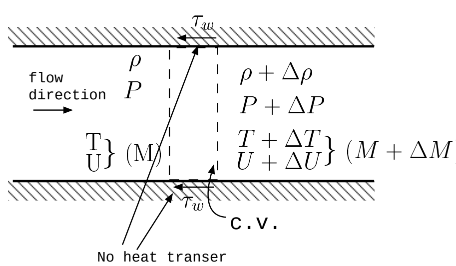


Fig. -A.1. Control volume of the gas flow in a constant cross section

This model explains many industrial flow processes which includes emptying of pressured container through a relatively short tube, exhaust system of an internal combustion engine, compressed air systems, etc. As this model raised from need to explain the steam flow in turbines.

A.1 Introduction

Consider a gas flowing through a conduit with a friction (see Figure (A.1)). It is advantages to examine the simplest situation and yet without losing the core properties of the process. Later, more general cases will be examined[2].

[1] Even the friction does not convert into heat

[2] Not ready yet, discussed on the ideal gas model and the entry length issues.

A.2 Fanno Model

The mass (continuity equation) balance can be written as

$$\dot{m} = \rho A U = constant \\ \hookrightarrow \rho_1 U_1 = \rho_2 U_2 \tag{A.1}$$

The energy conservation (under the assumption that this model is adiabatic flow and the friction is not transformed into thermal energy) reads

$$T_{0\,1} = T_{0\,2} \tag{A.2}$$

$$\hookrightarrow T_1 + \frac{{U_1}^2}{2c_p} = T_2 + \frac{{U_2}^2}{2c_p} \tag{A.3}$$

Or in a derivative from

$$C_p dT + d\left(\frac{U^2}{2}\right) = 0 \tag{A.4}$$

Again for simplicity, the perfect gas model is assumed[3].

$$P = \rho R T \\ \hookrightarrow \frac{P_1}{\rho_1 T_1} = \frac{P_2}{\rho_2 T_2} \tag{A.5}$$

It is assumed that the flow can be approximated as one–dimensional. The force acting on the gas is the friction at the wall and the momentum conservation reads

$$-AdP - \tau_w dA_w = \dot{m} dU \tag{A.6}$$

It is convenient to define a hydraulic diameter as

$$D_H = \frac{4 \times \text{Cross Section Area}}{\text{wetted perimeter}} \tag{A.7}$$

Or in other words

$$A = \frac{\pi {D_H}^2}{4} \tag{A.8}$$

[3]The equation of state is written again here so that all the relevant equations can be found when this chapter is printed separately.

It is convenient to substitute D for D_H and yet it still will be referred to the same name as the hydraulic diameter. The infinitesimal area that shear stress is acting on is

$$dA_w = \pi D dx \tag{A.9}$$

Introducing the Fanning friction factor as a dimensionless friction factor which is some times referred to as the friction coefficient and reads as the following:

$$f = \frac{\tau_w}{\frac{1}{2}\rho U^2} \tag{A.10}$$

By utilizing equation (A.2) and substituting equation (A.10) into momentum equation (A.6) yields

$$-\overbrace{\frac{\pi D^2}{4}}^{A} dP - \pi D dx \overbrace{f\left(\frac{1}{2}\rho U^2\right)}^{\tau_w} = A \overbrace{\rho U}^{\frac{\dot{m}}{A}} dU \tag{A.11}$$

Dividing equation (A.11) by the cross section area, $\mathbf{A}$ and rearranging yields

$$-dP + \frac{4 f dx}{D}\left(\frac{1}{2}\rho U^2\right) = \rho U dU \tag{A.12}$$

The second law is the last equation to be utilized to determine the flow direction.

$$s_2 \geq s_1 \tag{A.13}$$

A.3 Non–Dimensionalization of the Equations

Before solving the above equation a dimensionless process is applied. By utilizing the definition of the sound speed to produce the following identities for perfect gas

$$M^2 = \left(\frac{U}{c}\right)^2 = \frac{U^2}{k \underbrace{RT}_{\frac{P}{\rho}}} \tag{A.14}$$

Utilizing the definition of the perfect gas results in

$$M^2 = \frac{\rho U^2}{kP} \tag{A.15}$$

Using the identity in equation (A.14) and substituting it into equation (A.11) and after some rearrangement yields

$$-dP + \frac{4 f dx}{D_H}\left(\frac{1}{2} k P M^2\right) = \frac{\rho U^2}{U} dU = \overbrace{kPM^2}^{\rho U^2} \frac{dU}{U} \tag{A.16}$$

By further rearranging equation (A.16) results in

$$-\frac{dP}{P} - \frac{4fdx}{D}\left(\frac{kM^2}{2}\right) = kM^2\frac{dU}{U} \tag{A.17}$$

It is convenient to relate expressions of (dP/P) and dU/U in terms of the Mach number and substituting it into equation (A.17). Derivative of mass conservation ((A.2)) results in

$$\frac{d\rho}{\rho} + \overbrace{\frac{1}{2}\frac{dU^2}{U^2}}^{\frac{dU}{U}} = 0 \tag{A.18}$$

The derivation of the equation of state (A.5) and dividing the results by equation of state (A.5) results

$$\frac{dP}{P} = \frac{d\rho}{\rho} + \frac{dT}{dT} \tag{A.19}$$

Derivation of the Mach identity equation (A.14) and dividing by equation (A.14) yields

$$\frac{d(M^2)}{M^2} = \frac{d(U^2)}{U^2} - \frac{dT}{T} \tag{A.20}$$

Dividing the energy equation (A.4) by C_p and by utilizing the definition Mach number yields

$$\begin{aligned}
&\frac{dT}{T} + \underbrace{\frac{1}{\left(\frac{kR}{(k-1)}\right)}}_{C_p}\frac{1}{T}\frac{U^2}{U^2}d\left(\frac{U^2}{2}\right) = \\
&\hookrightarrow \frac{dT}{T} + \frac{(k-1)}{\underbrace{kRT}_{c^2}}\frac{U^2}{U^2}d\left(\frac{U^2}{2}\right) = \\
&\hookrightarrow \frac{dT}{T} + \frac{k-1}{2}M^2\frac{dU^2}{U^2} = 0
\end{aligned} \tag{A.21}$$

Equations (A.17), (A.18), (A.19), (A.20), and (A.21) need to be solved. These equations are separable so one variable is a function of only single variable (the chosen as the independent variable). Explicit explanation is provided for only two variables, the rest variables can be done in a similar fashion. The dimensionless friction, $\frac{4fL}{D}$, is chosen as the independent variable since the change in the dimensionless resistance, $\frac{4fL}{D}$, causes the change in the other variables.

Combining equations (A.19) and (A.21) when eliminating dT/T results

$$\frac{dP}{P} = \frac{d\rho}{\rho} - \frac{(k-1)M^2}{2}\frac{dU^2}{U^2} \tag{A.22}$$

The term $\frac{d\rho}{\rho}$ can be eliminated by utilizing equation (A.18) and substituting it into equation (A.22) and rearrangement yields

$$\frac{dP}{P} = -\frac{1+(k-1)M^2}{2}\frac{dU^2}{U^2} \tag{A.23}$$

The term dU^2/U^2 can be eliminated by using (A.23)

$$\frac{dP}{P} = -\frac{kM^2\left(1+(k-1)M^2\right)}{2(1-M^2)}\frac{4fdx}{D} \tag{A.24}$$

The second equation for Mach number, M variable is obtained by combining equation (A.20) and (A.21) by eliminating dT/T. Then $d\rho/\rho$ and U are eliminated by utilizing equation (A.18) and equation (A.22). The only variable that is left is P (or dP/P) which can be eliminated by utilizing equation (A.24) and results in

$$\frac{4fdx}{D} = \frac{\left(1-M^2\right)dM^2}{kM^4(1+\frac{k-1}{2}M^2)} \tag{A.25}$$

Rearranging equation (A.25) results in

$$\frac{dM^2}{M^2} = \frac{kM^2\left(1+\frac{k-1}{2}M^2\right)}{1-M^2}\frac{4fdx}{D} \tag{A.26}$$

After similar mathematical manipulation one can get the relationship for the velocity to read

$$\frac{dU}{U} = \frac{kM^2}{2\left(1-M^2\right)}\frac{4fdx}{D} \tag{A.27}$$

and the relationship for the temperature is

$$\frac{dT}{T} = \frac{1}{2}\frac{dc}{c} = -\frac{k(k-1)M^4}{2(1-M^2)}\frac{4fdx}{D} \tag{A.28}$$

density is obtained by utilizing equations (A.27) and (A.18) to obtain

$$\frac{d\rho}{\rho} = -\frac{kM^2}{2\left(1-M^2\right)}\frac{4fdx}{D} \tag{A.29}$$

The stagnation pressure is similarly obtained as

$$\frac{dP_0}{P_0} = -\frac{kM^2}{2}\frac{4fdx}{D} \tag{A.30}$$

The second law reads

$$ds = C_p \ln\frac{dT}{T} - R\ln\frac{dP}{P} \tag{A.31}$$

The stagnation temperature expresses as $T_0 = T(1+(1-k)/2M^2)$. Taking derivative of this expression when M remains constant yields $dT_0 = dT(1+(1-k)/2M^2)$ and thus when these equations are divided they yield

$$dT/T = dT_0/T_0 \tag{A.32}$$

In similar fashion the relationship between the stagnation pressure and the pressure can be substituted into the entropy equation and result in

$$ds = C_p \ln \frac{dT_0}{T_0} - R \ln \frac{dP_0}{P_0} \tag{A.33}$$

The first law requires that the stagnation temperature remains constant, $(dT_0 = 0)$. Therefore the entropy change is

$$\frac{ds}{C_p} = -\frac{(k-1)}{k}\frac{dP_0}{P_0} \tag{A.34}$$

Using the equation for stagnation pressure the entropy equation yields

$$\frac{ds}{C_p} = \frac{(k-1)M^2}{2}\frac{4fdx}{D} \tag{A.35}$$

A.4 The Mechanics and Why the Flow is Choked?

The trends of the properties can be examined by looking in equations (A.24) through (A.34). For example, from equation (A.24) it can be observed that the critical point is when $M = 1$. When $M < 1$ the pressure decreases downstream as can be seen from equation (A.24) because fdx and M are positive. For the same reasons, in the supersonic branch, $M > 1$, the pressure increases downstream. This pressure increase is what makes compressible flow so different from "conventional" flow. Thus the discussion will be divided into two cases: One, flow above speed of sound. Two, flow with speed below the speed of sound.

Why the flow is choked?

Here, the explanation is based on the equations developed earlier and there is no known explanation that is based on the physics. First, it has to be recognized that the critical point is when $M = 1$. It will be shown that a change in location relative to this point change the trend and it is singular point by itself. For example, $dP(@M = 1) = \infty$ and mathematically it is a singular point (see equation (A.24)). Observing from equation (A.24) that increase or decrease from subsonic just below one $M = (1-\epsilon)$ to above just above one $M = (1+\epsilon)$ requires a change in a sign pressure direction. However, the pressure has to be a monotonic function which means that flow cannot crosses over the point of $M = 1$. This constrain means that because the flow cannot "crossover" $M = 1$ the gas has to reach to this speed, $M = 1$ at the last point. This situation is called choked flow.

The Trends

The trends or whether the variables are increasing or decreasing can be observed from looking at the equation developed. For example, the pressure can be examined by looking at equation (A.26). It demonstrates that the Mach number increases downstream when the flow is subsonic. On the other hand, when the flow is supersonic, the pressure decreases.

The summary of the properties changes on the sides of the branch

	Subsonic	Supersonic
Pressure, P	decrease	increase
Mach number, M	increase	decrease
Velocity, U	increase	decrease
Temperature, T	decrease	increase
Density, ρ	decrease	increase
Stagnation Temperature, T_0	decrease	increase

A.5 The Working Equations

Integration of equation (A.25) yields

$$\boxed{\frac{4}{D}\int_{L}^{L_{max}} f dx = \frac{1}{k}\frac{1-M^2}{M^2} + \frac{k+1}{2k}\ln\frac{\frac{k+1}{2}M^2}{1+\frac{k-1}{2}M^2}} \tag{A.36}$$

A representative friction factor is defined as

$$\bar{f} = \frac{1}{L_{max}}\int_{0}^{L_{max}} f dx \tag{A.37}$$

In the isothermal flow model it was shown that friction factor is constant through the process if the fluid is ideal gas. Here, the Reynolds number defined in equation (**??**) is not constant because the temperature is not constant. The viscosity even for ideal gas is complex function of the temperature (further reading in "Basic of Fluid Mechanics" chapter one, Potto Project). However, the temperature variation is very limit. Simple improvement can be done by assuming constant constant viscosity (constant friction factor) and find the temperature on the two sides of the tube to improve the friction factor for the next iteration. The maximum error can be estimated by looking at the maximum change of the temperature. The temperature can be reduced by less than 20% for most range of the spesific heats ratio. The viscosity change for this change is for many gases about 10%. For these gases the maximum increase of average Reynolds number is only 5%. What this change in Reynolds number does to friction factor? That depend in the range of Reynolds number. For Reynolds number larger than 10,000 the change in friction factor can be considered negligible. For the other extreme, laminar flow it can estimated that change of 5% in Reynolds number change about the same amount in friction factor. With the exception the jump from a laminar flow to a turbulent flow, the change is noticeable but very small. In the light of the about discussion the friction factor is assumed to constant. By utilizing the mean average theorem equation (A.36) yields

$$\boxed{\frac{4\bar{f}L_{max}}{D} = \frac{1}{k}\frac{1-M^2}{M^2} + \frac{k+1}{2k}\ln\frac{\frac{k+1}{2}M^2}{1+\frac{k-1}{2}M^2}} \tag{A.38}$$

It is common to replace the $\bar{f}$ with f which is adopted in this book.

Equations (A.24), (A.27), (A.28), (A.29), (A.29), and (A.30) can be solved. For example, the pressure as written in equation (A.23) is represented by $\frac{4fL}{D}$, and Mach number. Now equation (A.24) can eliminate term $\frac{4fL}{D}$ and describe the pressure on the Mach number. Dividing equation (A.24) in equation (A.26) yields

$$\frac{\frac{dP}{P}}{\frac{dM^2}{M^2}} = -\frac{1+(k-1M^2}{2M^2\left(1+\frac{k-1}{2}M^2\right)}dM^2 \tag{A.39}$$

The symbol "*" denotes the state when the flow is choked and Mach number is equal to 1. Thus, $M=1$ when $P=P^*$ equation (A.39) can be integrated to yield:

$$\frac{P}{P^*} = \frac{1}{M}\sqrt{\frac{\frac{k+1}{2}}{1+\frac{k-1}{2}M^2}} \tag{A.40}$$

In the same fashion the variables ratio can be obtained

$$\frac{T}{T^*} = \frac{c^2}{{c^*}^2} = \frac{\frac{k+1}{2}}{1+\frac{k-1}{2}M^2} \tag{A.41}$$

$$\frac{\rho}{\rho^*} = \frac{1}{M}\sqrt{\frac{1+\frac{k-1}{2}M^2}{\frac{k+1}{2}}} \tag{A.42}$$

$$\frac{U}{U^*} = \left(\frac{\rho}{\rho^*}\right)^{-1} = M\sqrt{\frac{\frac{k+1}{2}}{1+\frac{k-1}{2}M^2}} \tag{A.43}$$

The stagnation pressure decreases and can be expressed by

$$\frac{P_0}{{P_0}^*} = \frac{\overbrace{\frac{P_0}{P}}^{\left(1+\frac{1-k}{2}M^2\right)^{\frac{k}{k-1}}} \quad P}{\underbrace{\frac{{P_0}^*}{P^*}}_{\left(\frac{2}{k+1}\right)^{\frac{k}{k-1}}} \quad P^*} \tag{A.44}$$

Using the pressure ratio in equation (A.40) and substituting it into equation (A.44) yields

$$\frac{P_0}{{P_0}^*} = \left(\frac{1+\frac{k-1}{2}M^2}{\frac{k+1}{2}}\right)^{\frac{k}{k-1}} \frac{1}{M}\sqrt{\frac{1+\frac{k-1}{2}M^2}{\frac{k+1}{2}}} \tag{A.45}$$

And further rearranging equation (A.45) provides

$$\frac{P_0}{{P_0}^*} = \frac{1}{M}\left(\frac{1+\frac{k-1}{2}M^2}{\frac{k+1}{2}}\right)^{\frac{k+1}{2(k-1)}} \tag{A.46}$$

The integration of equation (A.34) yields

$$\frac{s-s^*}{c_p} = \ln M^2 \sqrt{\left(\frac{k+1}{2M^2\left(1+\frac{k-1}{2}M^2\right)}\right)^{\frac{k+1}{k}}} \tag{A.47}$$

The results of these equations are plotted in Figure (A.2) The Fanno flow is in many cases shockless and therefore a relationship between two points should be derived. In most times, the "star" values are imaginary values that represent the value at choking. The real ratio can be obtained by two star ratios as an example

$$\frac{T_2}{T_1} = \frac{\left.\frac{T}{T^*}\right|_{M_2}}{\left.\frac{T}{T^*}\right|_{M_1}} \tag{A.48}$$

A special interest is the equation for the dimensionless friction as following

$$\int_{L_1}^{L_2} \frac{4fL}{D}dx = \int_{L_1}^{L_{max}} \frac{4fL}{D}dx - \int_{L_2}^{L_{max}} \frac{4fL}{D}dx \tag{A.49}$$

Hence,

$$\boxed{\left(\frac{4fL_{max}}{D}\right)_2 = \left(\frac{4fL_{max}}{D}\right)_1 - \frac{4fL}{D}} \tag{A.50}$$

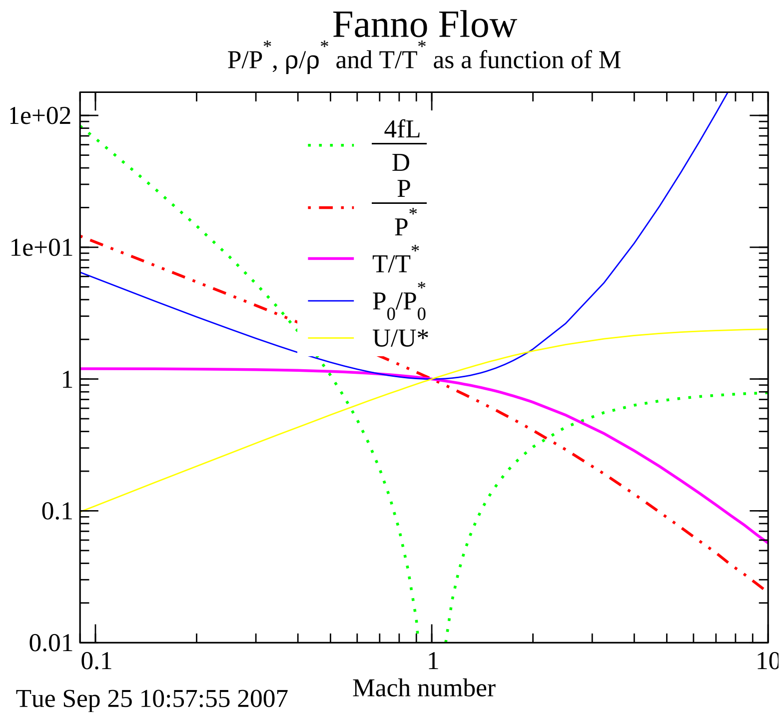


Fig. -A.2. Various parameters in Fanno flow as a function of Mach number

A.6 Examples of Fanno Flow

Example A.1:

Air flows from a reservoir and enters a uniform pipe with a diameter of 0.05 [m] and length of 10 [m]. The air exits to the atmosphere. The following conditions prevail at the exit: $P_2 = 1[bar]$ *temperature* $T_2 = 27°C$ $M_2 = 0.9$[4]. *Assume that the average friction factor to be* $f = 0.004$ *and that the flow from the reservoir up to the pipe inlet is essentially isentropic. Estimate the total temperature and total pressure in the reservoir under the Fanno flow model.*

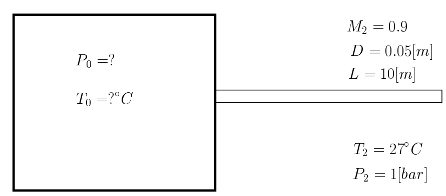


Fig. -A.3. Schematic of Example (A.1)

Solution

For isentropic, the flow to the pipe inlet, the temperature and the total pressure at the

[4]This property is given only for academic purposes. There is no Mach meter.

pipe inlet are the same as those in the reservoir. Thus, finding the total pressure and temperature at the pipe inlet is the solution. With the Mach number and temperature known at the exit, the total temperature at the entrance can be obtained by knowing the $\frac{4fL}{D}$. For given Mach number ($M = 0.9$) the following is obtained.

$\mathbf{M}$	$\mathbf{\frac{4fL}{D}}$	$\mathbf{\frac{P}{P^*}}$	$\mathbf{\frac{P_0}{{P_0}^*}}$	$\frac{\rho}{\rho^*}$	$\mathbf{\frac{U}{U^*}}$	$\mathbf{\frac{T}{T^*}}$
0.90000	0.01451	1.1291	1.0089	1.0934	0.9146	1.0327

So, the total temperature at the exit is

$$T^*|_2 = \left.\frac{T^*}{T}\right|_2 T_2 = \frac{300}{1.0327} = 290.5[K]$$

To "move" to the other side of the tube the $\frac{4fL}{D}$ is added as

$$\left.\tfrac{4fL}{D}\right|_1 = \tfrac{4fL}{D} + \left.\tfrac{4fL}{D}\right|_2 = \frac{4 \times 0.004 \times 10}{0.05} + 0.01451 \simeq 3.21$$

The rest of the parameters can be obtained with the new $\frac{4fL}{D}$ either from Table (A.1) by interpolations or by utilizing the attached program.

$\mathbf{M}$	$\mathbf{\frac{4fL}{D}}$	$\mathbf{\frac{P}{P^*}}$	$\mathbf{\frac{P_0}{{P_0}^*}}$	$\frac{\rho}{\rho^*}$	$\mathbf{\frac{U}{U^*}}$	$\mathbf{\frac{T}{T^*}}$
0.35886	3.2100	3.0140	1.7405	2.5764	0.38814	1.1699

Note that the subsonic branch is chosen. The stagnation ratios has to be added for $M = 0.35886$

$\mathbf{M}$	$\mathbf{\frac{T}{T_0}}$	$\frac{\rho}{\rho_0}$	$\mathbf{\frac{A}{A^\star}}$	$\mathbf{\frac{P}{P_0}}$	$\mathbf{\frac{A\times P}{A^*\times P_0}}$	$\mathbf{\frac{F}{F^*}}$
0.35886	0.97489	0.93840	1.7405	0.91484	1.5922	0.78305

The total pressure P_{01} can be found from the combination of the ratios as follows:

$$\begin{aligned}
P_{01} &= \overbrace{\overbrace{P_2 \left.\frac{P^*}{P}\right|_2}^{P^*} \left.\frac{P}{P^*}\right|_1}^{P_1} \left.\frac{P_0}{P}\right|_1 \\
&= 1 \times \frac{1}{1.12913} \times 3.014 \times \frac{1}{0.915} = 2.91[Bar]
\end{aligned}$$

$$
\begin{aligned}
T_{01} =& \overbrace{\overbrace{T_2 \left.\frac{T^*}{T}\right|_2}^{T^*} \left.\frac{T}{T^*}\right|_1}^{T_1} \left.\frac{T_0}{T}\right|_1 \\
=& 300 \times \frac{1}{1.0327} \times 1.17 \times \frac{1}{0.975} \simeq 348K = 75°\text{C}
\end{aligned}
$$

End Solution

Another academic question/example:

Example A.2:

A system is composed of a convergent-divergent nozzle followed by a tube with length of 2.5 [cm] in diameter and 1.0 [m] long. The system is supplied by a vessel. The vessel conditions are at 29.65 [Bar], 400 K. With these conditions a pipe inlet Mach number is 3.0. A normal shock wave occurs in the tube and the flow discharges to the atmosphere, determine:

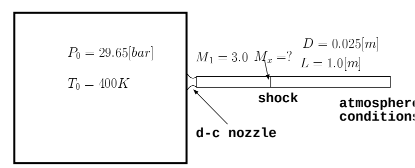


Fig. -A.4. The schematic of Example (A.2)

(a) the mass flow rate through the system;

(b) the temperature at the pipe exit; and

(c) determine the Mach number when a normal shock wave occurs [M_x].

Take k = 1.4, R = 287 $[J/kgK]$ and $f = 0.005$.

Solution

(a) Assuming that the pressure vessel is very much larger than the pipe, therefore the velocity in the vessel can be assumed to be small enough so it can be neglected. Thus, the stagnation conditions can be approximated for the condition in the tank. It is further assumed that the flow through the nozzle can be approximated as isentropic. Hence, $T_{01} = 400K$ and $P_{01} = 29.65[Par]$

The mass flow rate through the system is constant and for simplicity point 1 is chosen in which,

$$\dot{m} = \rho A M c$$

The density and speed of sound are unknowns and need to be computed. With the isentropic relationship the Mach number at point one (1) is known, then the following can be found either from Table (A.1) or the Potto–GDC

$\mathbf{M}$	$\frac{\mathbf{T}}{\mathbf{T_0}}$	$\frac{\rho}{\rho_0}$	$\frac{\mathbf{A}}{\mathbf{A^\star}}$	$\frac{\mathbf{P}}{\mathbf{P_0}}$	$\frac{\mathbf{A\times P}}{\mathbf{A^*\times P_0}}$	$\frac{\mathbf{F}}{\mathbf{F^*}}$
3.0000	0.35714	0.07623	4.2346	0.02722	0.11528	0.65326

The temperature is

$$T_1 = \frac{T_1}{T_{01}} T_{01} = 0.357 \times 400 = 142.8K$$

Using the temperature, the speed of sound can be calculated as

$$c_1 = \sqrt{kRT} = \sqrt{1.4 \times 287 \times 142.8} \simeq 239.54[m/sec]$$

The pressure at point 1 can be calculated as

$$P_1 = \frac{P_1}{P_{01}} P_{01} = 0.027 \times 30 \simeq 0.81[Bar]$$

The density as a function of other properties at point 1 is

$$\rho_1 = \left.\frac{P}{RT}\right|_1 = \frac{8.1 \times 10^4}{287 \times 142.8} \simeq 1.97 \left[\frac{kg}{m^3}\right]$$

The mass flow rate can be evaluated from equation (A.2)

$$\dot{m} = 1.97 \times \frac{\pi \times 0.025^2}{4} \times 3 \times 239.54 = 0.69 \left[\frac{kg}{sec}\right]$$

(b) First, check whether the flow is shockless by comparing the flow resistance and the maximum possible resistance. From the Table (A.1) or by using the Potto–GDC, to obtain the following

$\mathbf{M}$	$\frac{\mathbf{4fL}}{\mathbf{D}}$	$\frac{\mathbf{P}}{\mathbf{P^*}}$	$\frac{\mathbf{P_0}}{\mathbf{P_0}^*}$	$\frac{\rho}{\rho^*}$	$\frac{\mathbf{U}}{\mathbf{U^*}}$	$\frac{\mathbf{T}}{\mathbf{T^*}}$
3.0000	0.52216	0.21822	4.2346	0.50918	1.9640	0.42857

and the conditions of the tube are

$$\frac{4fL}{D} = \frac{4 \times 0.005 \times 1.0}{0.025} = 0.8$$

Since $0.8 > 0.52216$ the flow is choked and with a shock wave.

The exit pressure determines the location of the shock, if a shock exists, by comparing "possible" P_{exit} to P_B. Two possibilities are needed to be checked; one, the shock at the entrance of the tube, and two, shock at the exit and comparing the pressure ratios. First, the possibility that the shock wave occurs immediately at the entrance for which the ratio for M_x are (shock wave Table (**??**))

$\mathbf{M_x}$	$\mathbf{M_y}$	$\frac{\mathbf{T_y}}{\mathbf{T_x}}$	$\frac{\rho_y}{\rho_x}$	$\frac{\mathbf{P_y}}{\mathbf{P_x}}$	$\frac{\mathbf{P_{0_y}}}{\mathbf{P_{0_x}}}$
3.0000	0.47519	2.6790	3.8571	10.3333	0.32834

After the shock wave the flow is subsonic with "M_1"$= 0.47519$. (Fanno flow Table (A.1))

$\mathbf{M}$	$\frac{\mathbf{4fL}}{\mathbf{D}}$	$\frac{\mathbf{P}}{\mathbf{P^*}}$	$\frac{\mathbf{P_0}}{\mathbf{P_0{}^*}}$	$\frac{\rho}{\rho^*}$	$\frac{\mathbf{U}}{\mathbf{U^*}}$	$\frac{\mathbf{T}}{\mathbf{T^*}}$
0.47519	1.2919	2.2549	1.3904	1.9640	0.50917	1.1481

The stagnation values for $M = 0.47519$ are

$\mathbf{M}$	$\frac{\mathbf{T}}{\mathbf{T_0}}$	$\frac{\rho}{\rho_0}$	$\frac{\mathbf{A}}{\mathbf{A^*}}$	$\frac{\mathbf{P}}{\mathbf{P_0}}$	$\frac{\mathbf{A \times P}}{\mathbf{A^* \times P_0}}$	$\frac{\mathbf{F}}{\mathbf{F^*}}$
0.47519	0.95679	0.89545	1.3904	0.85676	1.1912	0.65326

The ratio of exit pressure to the chamber total pressure is

$$\begin{aligned}
\frac{P_2}{P_0} &= \overbrace{\left(\frac{P_2}{P^*}\right)}^{1}\left(\frac{P^*}{P_1}\right)\left(\frac{P_1}{P_{0y}}\right)\left(\frac{P_{0y}}{P_{0x}}\right)\overbrace{\left(\frac{P_{0x}}{P_0}\right)}^{1} \\
&= 1 \times \frac{1}{2.2549} \times 0.8568 \times 0.32834 \times 1 \\
&= 0.12476
\end{aligned}$$

The actual pressure ratio $1/29.65 = 0.0338$ is smaller than the case in which shock occurs at the entrance. Thus, the shock is somewhere downstream. One possible way to find the exit temperature, T_2 is by finding the location of the shock. To find the location of the shock ratio of the pressure ratio, $\frac{P_2}{P_1}$ is needed. With the location of shock, "claiming" upstream from the exit through shock to the entrance. For example, calculate the parameters for shock location with known $\frac{4fL}{D}$ in the "y" side. Then either by utilizing shock table or the program, to obtain the upstream Mach number.

The procedure for the calculations:

1) Calculate the entrance Mach number assuming the shock occurs at the exit:
   a) set $M_2^{'} = 1$ assume the flow in the entire tube is supersonic:
   b) calculated $M_1^{'}$
   Note this Mach number is the high Value.
2) Calculate the entrance Mach assuming shock at the entrance.
   a) set $M_2 = 1$
   b) add $\frac{4fL}{D}$ and calculated M_1' for subsonic branch
   c) calculated M_x for M_1'
   Note this Mach number is the low Value.
3) According your root finding algorithm[5] calculate or guess the shock location and then compute as above the new M_1.
   a) set $M_2 = 1$
   b) for the new $\frac{4fL}{D}$ and compute the new M_y' for the subsonic branch
   c) calculated M_x' for the M_y'
   d) Add the leftover of $\frac{4fL}{D}$ and calculated the M_1
4) guess new location for the shock according to your finding root procedure and according to the result, repeat previous stage until the solution is obtained.

$\mathbf{M_1}$	$\mathbf{M_2}$	$\left.\frac{\mathbf{4fL}}{\mathbf{D}}\right\|_{\mathbf{up}}$	$\left.\frac{\mathbf{4fL}}{\mathbf{D}}\right\|_{\mathbf{down}}$	$\mathbf{M_x}$	$\mathbf{M_y}$
3.0000	1.0000	0.22019	0.57981	1.9899	0.57910

(c) The way of the numerical procedure for solving this problem is by finding $\left.\frac{4fL}{D}\right|_{up}$ that will produce $M_1 = 3$. In the process M_x and M_y must be calculated (see the chapter on the program with its algorithms.).

End Solution

A.7 Supersonic Branch

In Chapter (**??**) it was shown that the isothermal model cannot describe adequately the situation because the thermal entry length is relatively large compared to the pipe length and the heat transfer is not sufficient to maintain constant temperature. In the Fanno model there is no heat transfer, and, furthermore, because the very limited amount of heat transformed it is closer to an adiabatic flow. The only limitation of the model is its uniform velocity (assuming parabolic flow for laminar and different profile for turbulent flow.). The information from the wall to the tube center[6] is slower in reality. However, experiments from many starting with 1938 work by Frossel[7] has shown that the error is not significant. Nevertheless, the comparison with reality shows that heat transfer cause changes to the flow and they need/should to be expected. These changes include the choking point at lower Mach number.

A.8 Maximum Length for the Supersonic Flow

It has to be noted and recognized that as opposed to subsonic branch the supersonic branch has a limited length. It also must be recognized that there is a maximum length for which only supersonic flow can exist[8]. These results were obtained from the mathematical derivations but were verified by numerous experiments[9]. The maximum length of the supersonic can be evaluated when $M = \infty$ as follows:

$$
\begin{aligned}
\frac{4fL_{max}}{D} &= \frac{1-M^2}{kM^2} + \frac{k+1}{2k} \ln \frac{\frac{k+1}{2}M^2}{2\left(1+\frac{k-1}{2}M^2\right)} = \\
\tfrac{4fL}{D}\left(M \to \infty\right) &\sim \frac{-\infty}{k \times \infty} + \frac{k+1}{2k} \ln \frac{(k+1)\infty}{(k-1)\infty} \\
&= \frac{-1}{k} + \frac{k+1}{2k} \ln \frac{(k+1)}{(k-1)} \\
&= \tfrac{4fL}{D}(M \to \infty, k = 1.4) = 0.8215
\end{aligned}
$$

The maximum length of the supersonic flow is limited by the above number. From the above analysis, it can be observed that no matter how high the entrance Mach number will be the tube length is limited and depends only on specific heat ratio, k as shown in Figure (A.5).

A.9 Working Conditions

It has to be recognized that there are two regimes that can occur in Fanno flow model one of subsonic flow and the other supersonic flow. Even the flow in the tube starts as a supersonic in parts of the tube can be transformed into the subsonic branch. A shock wave can occur and some portions of the tube will be in a subsonic flow pattern.

[6] The word information referred to is the shear stress transformed from the wall to the center of the tube.

[7] See on the web `http://naca.larc.nasa.gov/digidoc/report/tm/44/NACA-TM-844.PDF`

[8] Many in the industry have difficulties in understanding this concept. The author seeks for a nice explanation of this concept for non–fluid mechanics engineers. This solicitation is about how to explain this issue to non-engineers or engineer without a proper background.

[9] If you have experiments demonstrating this point, please provide to the undersign so they can be added to this book. Many of the pictures in the literature carry copyright statements.

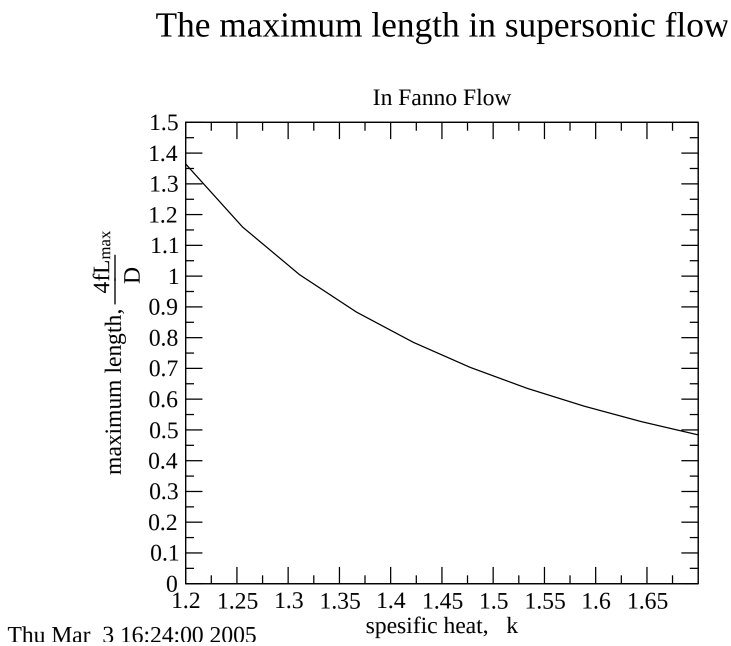


Fig. -A.5. The maximum length as a function of specific heat, k

The discussion has to differentiate between two ways of feeding the tube: converging nozzle or a converging-diverging nozzle. Three parameters, the dimensionless friction, $\frac{4fL}{D}$, the entrance Mach number, M_1, and the pressure ratio, P_2/P_1 are controlling the flow. Only a combination of these two parameters is truly independent. However, all the three parameters can be varied and they are discussed separately here.

A.9.1 Variations of The Tube Length ($\frac{4fL}{D}$) Effects

In the analysis of this effect, it should be assumed that back pressure is constant and/or low as possible as needed to maintain a choked flow. First, the treatment of the two branches are separated.

Fanno Flow Subsonic branch

For converging nozzle feeding, increasing the tube length results in increasing the exit Mach number (normally denoted herein as M_2). Once the Mach number reaches maximum ($M = 1$), no further increase of the exit Mach number can be achieved. In this process, the mass flow rate decreases. It is worth noting that entrance Mach number is reduced (as some might explain it to reduce the flow rate). The entrance temperature increases as can be seen from Figure (A.7). The velocity therefore must decrease because the loss of the enthalpy (stagnation temperature) is "used." The density decrease

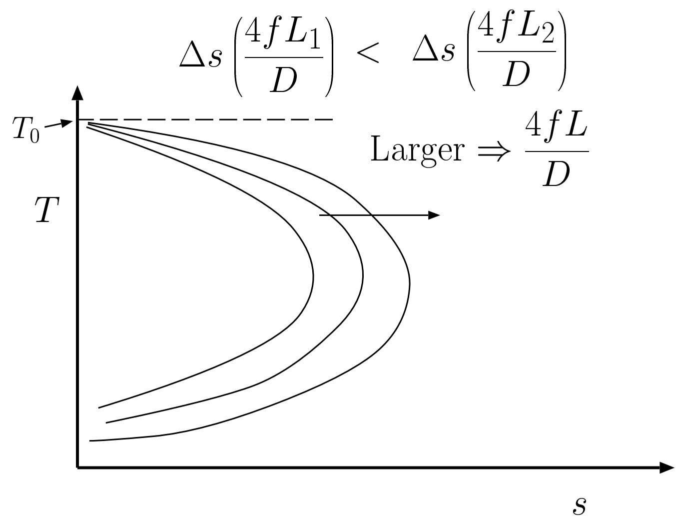


Fig. -A.6. The effects of increase of $\frac{4fL}{D}$ on the Fanno line

because $\rho = \frac{P}{RT}$ and when pressure is remains almost constant the density decreases. Thus, the mass flow rate must decrease. These results are applicable to the converging nozzle.

In the case of the converging–diverging feeding nozzle, increase of the dimensionless friction, $\frac{4fL}{D}$, results in a similar flow pattern as in the converging nozzle. Once the flow becomes choked a different flow pattern emerges.

Fanno Flow Supersonic Branch

There are several transitional points that change the pattern of the flow. Point $\mathbf{a}$ is the choking point (for the supersonic branch) in which the exit Mach number reaches to one. Point $\mathbf{b}$ is the maximum possible flow for supersonic flow and is not dependent on the nozzle. The next point, referred here as the critical point $\mathbf{c}$, is the point in which no supersonic flow is possible in the tube i.e. the shock reaches to the nozzle. There is another point $\mathbf{d}$, in which no supersonic flow is possible in the entire nozzle–tube system. Between these transitional points the effect parameters such as mass flow rate, entrance and exit Mach number are discussed.

At the starting point the flow is choked in the nozzle, to achieve supersonic flow. The following ranges that has to be discussed includes (see Figure (A.8)):

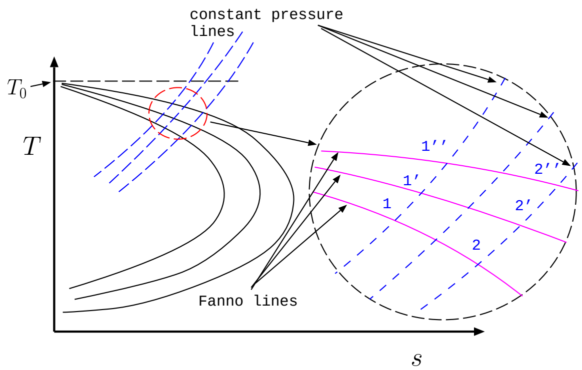


Fig. -A.7. The development properties in of converging nozzle

$$
\begin{array}{rcccll}
0 & < & \frac{4fL}{D} & < & \left(\frac{4fL}{D}\right)_{choking} & 0 \rightarrow \mathbf{a} \\
\left(\frac{4fL}{D}\right)_{choking} & < & \frac{4fL}{D} & < & \left(\frac{4fL}{D}\right)_{shockless} & \mathbf{a} \rightarrow \mathbf{b} \\
\left(\frac{4fL}{D}\right)_{shockless} & < & \frac{4fL}{D} & < & \left(\frac{4fL}{D}\right)_{chokeless} & \mathbf{b} \rightarrow \mathbf{c} \\
\left(\frac{4fL}{D}\right)_{chokeless} & < & \frac{4fL}{D} & < & \infty & \mathbf{c} \rightarrow \infty
\end{array}
$$

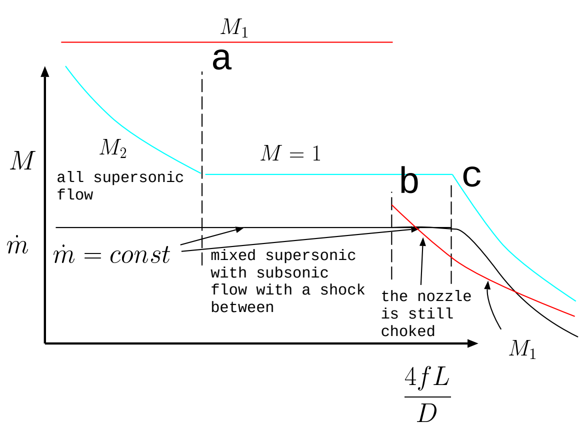


Fig. -A.8. The Mach numbers at entrance and exit of tube and mass flow rate for Fanno Flow as a function of the $\frac{4fL}{D}$.

The 0-$\mathbf{a}$ range, the mass flow rate is constant because the flow is choked at the nozzle. The entrance Mach number, M_1 is constant because it is a function of the nozzle design only. The exit Mach number, M_2 decreases (remember this flow is on the supersonic branch) and starts ($\frac{4fL}{D} = 0$) as $M_2 = M_1$. At the end of the range $\mathbf{a}$, $M_2 = 1$. In the range of $\mathbf{a} - \mathbf{b}$ the flow is all supersonic.

In the next range $\mathbf{a} - -\mathbf{b}$ The flow is double choked and make the adjustment for the flow rate at different choking points by changing the shock location. The mass flow rate continues to be constant. The entrance Mach continues to be constant and exit Mach number is constant.

The total maximum available for supersonic flow $\mathbf{b} - -\mathbf{b}'$, $\left(\frac{4fL}{D}\right)_{max}$, is only a theoretical length in which the supersonic flow can occur if nozzle is provided with a larger Mach number (a change to the nozzle area ratio which also reduces the mass flow rate). In the range $\mathbf{b} - \mathbf{c}$, it is a more practical point.

In semi supersonic flow $\mathbf{b} - \mathbf{c}$ (in which no supersonic is available in the tube but only in the nozzle) the flow is still double choked and the mass flow rate is constant. Notice that exit Mach number, M_2 is still one. However, the entrance Mach number, M_1, reduces with the increase of $\frac{4fL}{D}$.

It is worth noticing that in the $\mathbf{a} - \mathbf{c}$ the mass flow rate nozzle entrance velocity and the exit velocity remains constant![10]

In the last range $\mathbf{c} - \infty$ the end is really the pressure limit or the break of the model and the isothermal model is more appropriate to describe the flow. In this range, the flow rate decreases since $(\dot{m} \propto M_1)$[11].

To summarize the above discussion, Figures (A.8) exhibits the development of M_1, M_2 mass flow rate as a function of $\frac{4fL}{D}$. Somewhat different then the subsonic branch the mass flow rate is constant even if the flow in the tube is completely subsonic. This situation is because of the "double" choked condition in the nozzle. The exit Mach M_2 is a continuous monotonic function that decreases with $\frac{4fL}{D}$. The entrance Mach M_1 is a non continuous function with a jump at the point when shock occurs at the entrance "moves" into the nozzle.

Figure (A.9) exhibits the M_1 as a function of M_2. The Figure was calculated by utilizing the data from Figure (A.2) by obtaining the $\left.\frac{4fL}{D}\right|_{max}$ for M_2 and subtracting the given $\frac{4fL}{D}$ and finding the corresponding M_1.

The Figure (A.10) exhibits the entrance Mach number as a function of the M_2. Obviously there can be two extreme possibilities for the subsonic exit branch. Subsonic velocity occurs for supersonic entrance velocity, one, when the shock wave occurs at the tube exit and two, at the tube entrance. In Figure (A.10) only for $\frac{4fL}{D} = 0.1$ and $\frac{4fL}{D} = 0.4$ two extremes are shown. For $\frac{4fL}{D} = 0.2$ shown with only shock at the exit only. Obviously, and as can be observed, the larger $\frac{4fL}{D}$ creates larger differences between exit Mach number for the different shock locations. The larger $\frac{4fL}{D}$ larger M_1 must occurs even for shock at the entrance.

For a given $\frac{4fL}{D}$, below the maximum critical length, the supersonic entrance flow has three different regimes which depends on the back pressure. One, shockless flow,

[10]On a personal note, this situation is rather strange to explain. On one hand, the resistance increases and on the other hand, the exit Mach number remains constant and equal to one. Does anyone have an explanation for this strange behavior suitable for non–engineers or engineers without background in fluid mechanics?

[11]Note that ρ_1 increases with decreases of M_1 but this effect is less significant.

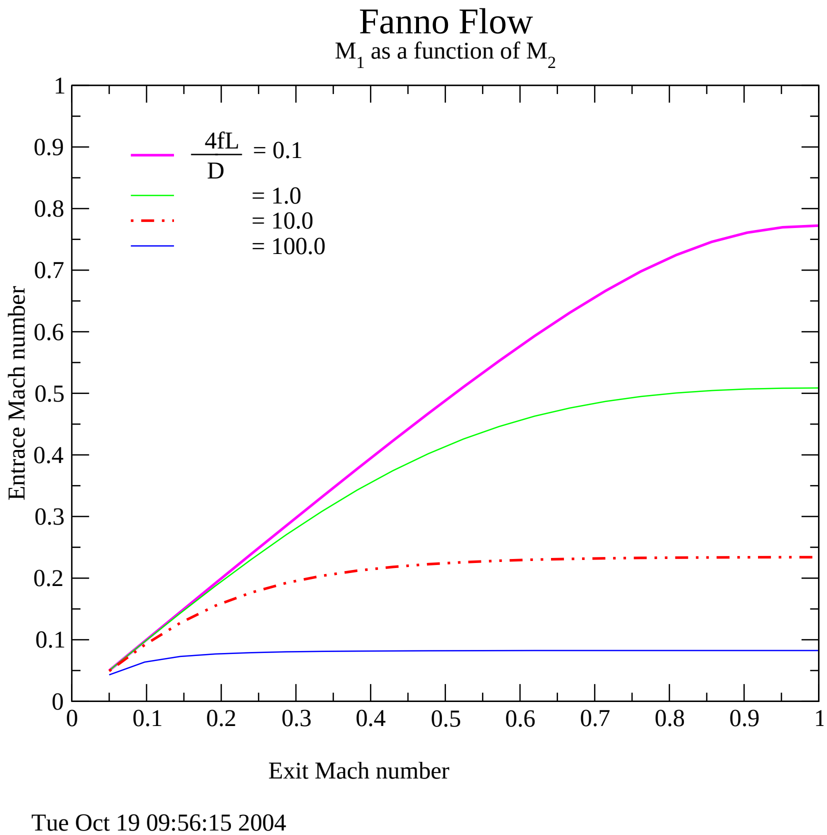


Fig. -A.9. M_1 as a function M_2 for various $\frac{4fL}{D}$

tow, shock at the entrance, and three, shock at the exit. Below, the maximum critical length is mathematically

$$\frac{4fL}{D} > -\frac{1}{k} + \frac{1+k}{2k} \ln \frac{k+1}{k-1}$$

For cases of $\frac{4fL}{D}$ above the maximum critical length no supersonic flow can be over the whole tube and at some point a shock will occur and the flow becomes subsonic flow[12].

A.9.2 The Pressure Ratio, $\frac{P_2}{P_1}$, effects

In this section the studied parameter is the variation of the back pressure and thus, the pressure ratio $\frac{P_2}{P_1}$ variations. For very low pressure ratio the flow can be assumed as incompressible with exit Mach number smaller than < 0.3. As the pressure ratio

[12]See more on the discussion about changing the length of the tube.

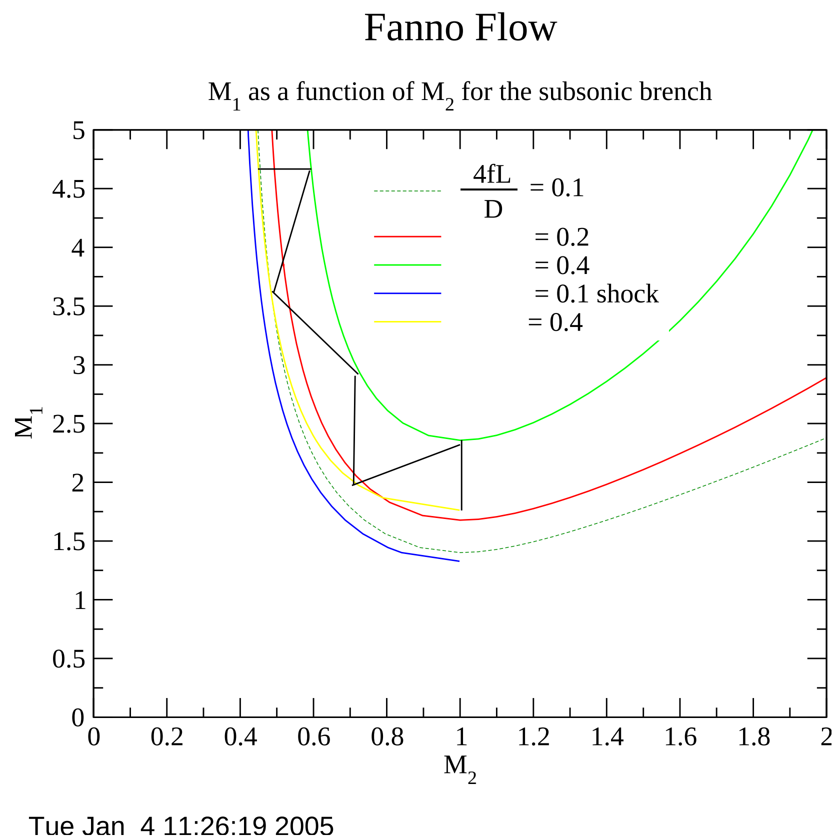


Fig. -A.10. M_1 as a function M_2 for different $\frac{4fL}{D}$ for supersonic entrance velocity.

increases (smaller back pressure, P_2), the exit and entrance Mach numbers increase. According to Fanno model the value of $\frac{4fL}{D}$ is constant (friction factor, f, is independent of the parameters such as, Mach number, Reynolds number et cetera) thus the flow remains on the same Fanno line. For cases where the supply come from a reservoir with a constant pressure, the entrance pressure decreases as well because of the increase in the entrance Mach number (velocity).

Again a differentiation of the feeding is important to point out. If the feeding nozzle is converging than the flow will be only subsonic. If the nozzle is "converging–diverging" than in some part supersonic flow is possible. At first the converging nozzle is presented and later the converging-diverging nozzle is explained.

Choking explanation for pressure variation/reduction

Decreasing the pressure ratio or in actuality the back pressure, results in increase of the entrance and the exit velocity until a maximum is reached for the exit velocity.

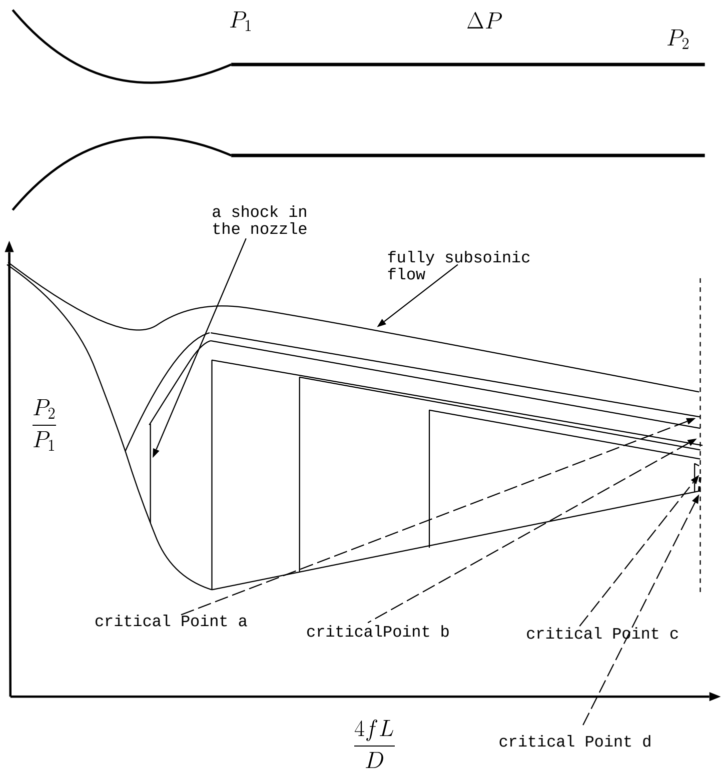


Fig. -A.11. The pressure distribution as a function of $\frac{4fL}{D}$ for a short $\frac{4fL}{D}$

The maximum velocity is when exit Mach number equals one. The Mach number, as it was shown in Chapter (**??**), can increases only if the area increase. In our model the tube area is postulated as a constant therefore the velocity cannot increase any further. However, for the flow to be continuous the pressure must decrease and for that the velocity must increase. Something must break since there are conflicting demands and it result in a "jump" in the flow. This jump is referred to as a choked flow. Any additional reduction in the back pressure will not change the situation in the tube. The only change will be at tube surroundings which are irrelevant to this discussion.

If the feeding nozzle is a "converging–diverging" then it has to be differentiated between two cases; One case is where the $\frac{4fL}{D}$ is short or equal to the critical length. The critical length is the maximum $\left.\frac{4fL}{D}\right|_{max}$ that associate with entrance Mach number.

Short $\frac{4fL}{D}$

Figure (A.12) shows different pressure profiles for different back pressures. Before the flow reaches critical point a (in the Figure) the flow is subsonic. Up to this stage the

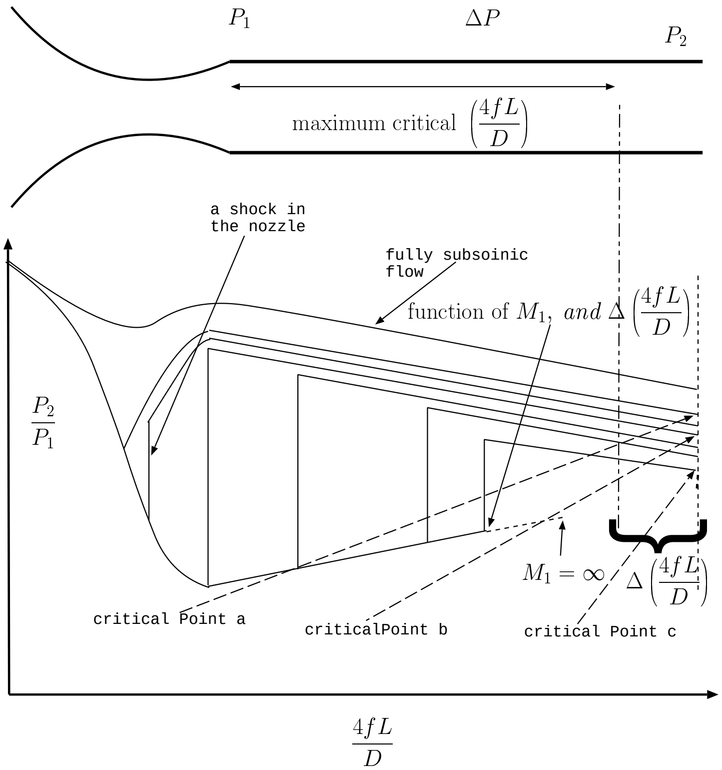


Fig. -A.12. The pressure distribution as a function of $\frac{4fL}{D}$ for a long $\frac{4fL}{D}$

nozzle feeding the tube increases the mass flow rate (with decreasing back pressure). Between point a and point b the shock is in the nozzle. In this range and further reduction of the pressure the mass flow rate is constant no matter how low the back pressure is reduced. Once the back pressure is less than point b the supersonic reaches to the tube. Note however that exit Mach number, $M_2 < 1$ and is **not** 1. A back pressure that is at the critical point c results in a shock wave that is at the exit. When the back pressure is below point c, the tube is "clean" of any shock[13]. The back pressure below point c has some adjustment as it occurs with exceptions of point d.

Long $\frac{4fL}{D}$

In the case of $\frac{4fL}{D} > \left.\frac{4fL}{D}\right|_{max}$ reduction of the back pressure results in the same process as explained in the short $\frac{4fL}{D}$ up to point c. However, point c in this case is different from point c at the case of short tube $\frac{4fL}{D} < \left.\frac{4fL}{D}\right|_{max}$. In this point the

[13]It is common misconception that the back pressure has to be at point d.

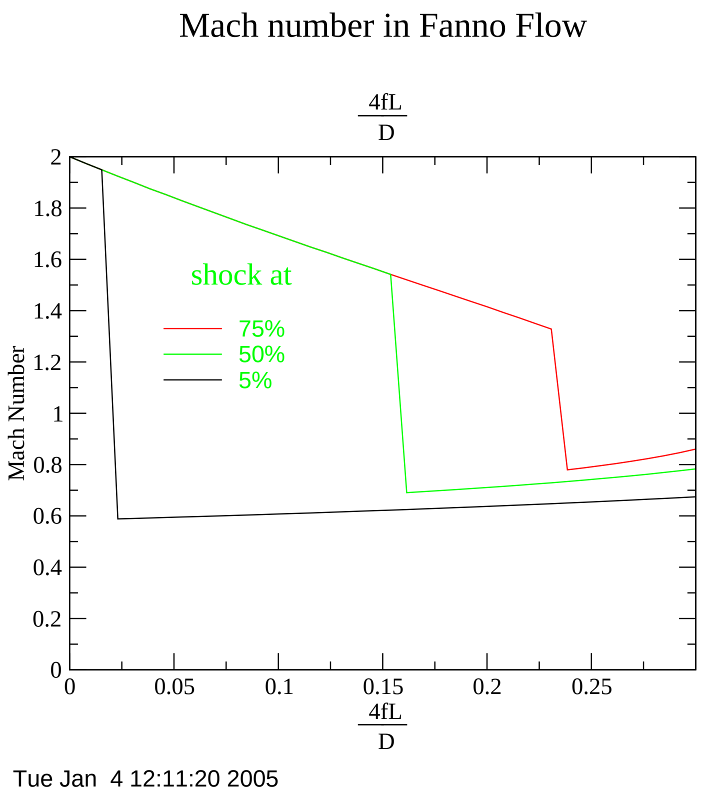


Fig. -A.13. The effects of pressure variations on Mach number profile as a function of $\frac{4fL}{D}$ when the total resistance $\frac{4fL}{D} = 0.3$ for Fanno Flow

exit Mach number is equal to 1 and the flow is double shock. Further reduction of the back pressure at this stage will not "move" the shock wave downstream the nozzle. At point c or location of the shock wave, is a function entrance Mach number, M_1 and the "extra" $\frac{4fL}{D}$. The is no analytical solution for the location of this point c. The procedure is (will be) presented in later stage.

A.9.3 Entrance Mach number, M_1, effects

In this discussion, the effect of changing the throat area on the nozzle efficiency is neglected. In reality these effects have significance and needs to be accounted for some instances. This dissection deals only with the flow when it reaches the supersonic branch reached otherwise the flow is subsonic with regular effects. It is assumed that in this discussion that the pressure ratio $\frac{P_2}{P_1}$ is large enough to create a choked flow and $\frac{4fL}{D}$ is small enough to allow it to happen.

The entrance Mach number, M_1 is a function of the ratio of the nozzle's throat area to the nozzle exit area and its efficiency. This effect is the third parameter discussed

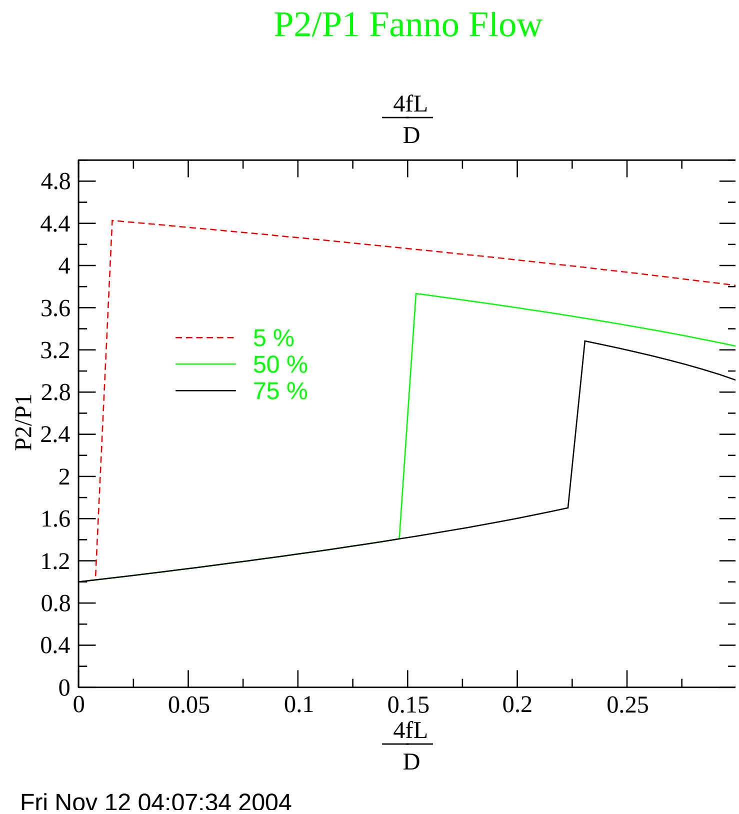


Fig. -A.14. Mach number as a function of $\frac{4fL}{D}$ *when the total* $\frac{4fL}{D} = 0.3$

here. Practically, the nozzle area ratio is changed by changing the throat area.

As was shown before, there are two different maximums for $\frac{4fL}{D}$; first is the total maximum $\frac{4fL}{D}$ of the supersonic which depends only on the specific heat, k, and second the maximum depends on the entrance Mach number, M_1. This analysis deals with the case where $\frac{4fL}{D}$ is shorter than total $\left.\frac{4fL}{D}\right|_{max}$.

Obviously, in this situation, the critical point is where $\frac{4fL}{D}$ is equal to $\left.\frac{4fL}{D}\right|_{max}$ as a result in the entrance Mach number.

The process of decreasing the converging–diverging nozzle's throat increases the entrance[14] Mach number. If the tube contains no supersonic flow then reducing the nozzle throat area wouldn't increase the entrance Mach number.

This part is for the case where some part of the tube is under supersonic regime and there is shock as a transition to subsonic branch. Decreasing the nozzle throat area

[14]The word "entrance" referred to the tube and not to the nozzle. The reference to the tube is because it is the focus of the study.

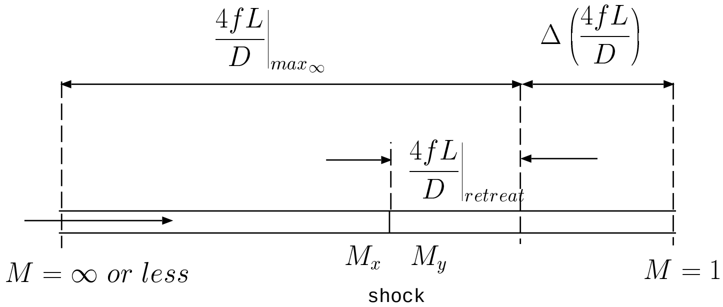


Fig. -A.15. Schematic of a "long" tube in supersonic branch

moves the shock location downstream. The "payment" for increase in the supersonic length is by reducing the mass flow. Further, decrease of the throat area results in flushing the shock out of the tube. By doing so, the throat area decreases. The mass flow rate is proportionally linear to the throat area and therefore the mass flow rate reduces. The process of decreasing the throat area also results in increasing the pressure drop of the nozzle (larger resistance in the nozzle[15])[16].

In the case of large tube $\frac{4fL}{D} > \left.\frac{4fL}{D}\right|_{max}$ the exit Mach number increases with the decrease of the throat area. Once the exit Mach number reaches one no further increases is possible. However, the location of the shock wave approaches to the theoretical location if entrance Mach, $M_1 = \infty$.

The maximum location of the shock The main point in this discussion however, is to find the furthest shock location downstream. Figure (A.16) shows the possible $\Delta\left(\frac{4fL}{D}\right)$ as function of retreat of the location of the shock wave from the maximum location. When the entrance Mach number is infinity, $M_1 = \infty$, if the shock location is at the maximum length, then shock at $M_x = 1$ results in $M_y = 1$.

The proposed procedure is based on Figure (A.16).

i) Calculate the extra $\frac{4fL}{D}$ and subtract the actual extra $\frac{4fL}{D}$ assuming shock at the left side (at the max length).

ii) Calculate the extra $\frac{4fL}{D}$ and subtract the actual extra $\frac{4fL}{D}$ assuming shock at the right side (at the entrance).

iii) According to the positive or negative utilizes your root finding procedure.

---

[15]Strange? Frictionless nozzle has a larger resistance when the throat area decreases

[16]It is one of the strange phenomenon that in one way increasing the resistance (changing the throat area) decreases the flow rate while in a different way (increasing the $\frac{4fL}{D}$) does not affect the flow rate.

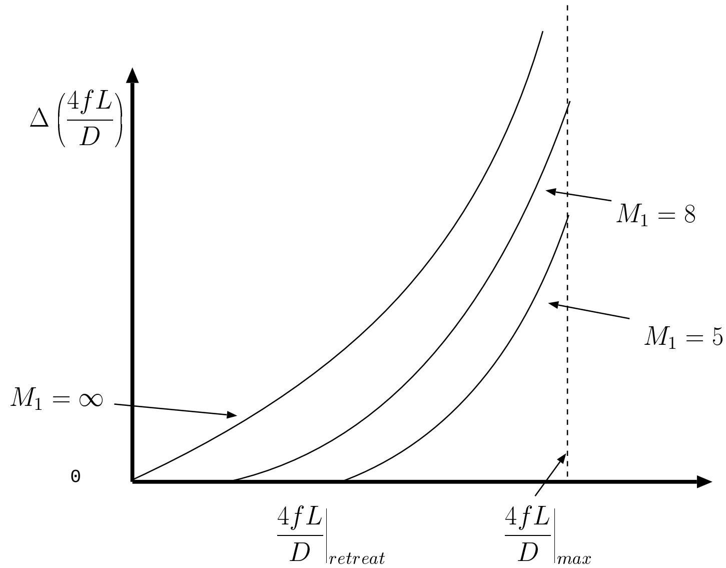


Fig. -A.16. The extra tube length as a function of the shock location, $\frac{4fL}{D}$ supersonic branch

From numerical point of view, the Mach number equal infinity when left side assumes result in infinity length of possible extra (the whole flow in the tube is subsonic). To overcome this numerical problem it is suggested to start the calculation from ϵ distance from the right hand side.

Let denote

$$\Delta\left(\frac{4fL}{D}\right) = \frac{4\bar{f}L}{D}_{actual} - \left.\frac{4fL}{D}\right|_{sup} \tag{A.51}$$

Note that $\left.\frac{4fL}{D}\right|_{sup}$ is smaller than $\left.\frac{4fL}{D}\right|_{max_\infty}$. The requirement that has to be satisfied is that denote $\left.\frac{4fL}{D}\right|_{retreat}$ as difference between the maximum possible of length in which the supersonic flow is achieved and the actual length in which the flow is supersonic see Figure (A.15). The retreating length is expressed as subsonic but

$$\left.\frac{4fL}{D}\right|_{retreat} = \left.\frac{4fL}{D}\right|_{max_\infty} - \left.\frac{4fL}{D}\right|_{sup} \tag{A.52}$$

Figure (A.17) shows the entrance Mach number, M_1 reduces after the maximum length is exceeded.

Example A.3:

Calculate the shock location for entrance Mach number $M_1 = 8$ and for $\frac{4fL}{D} = 0.9$ assume that $k = 1.4$ ($M_{exit} = 1$).

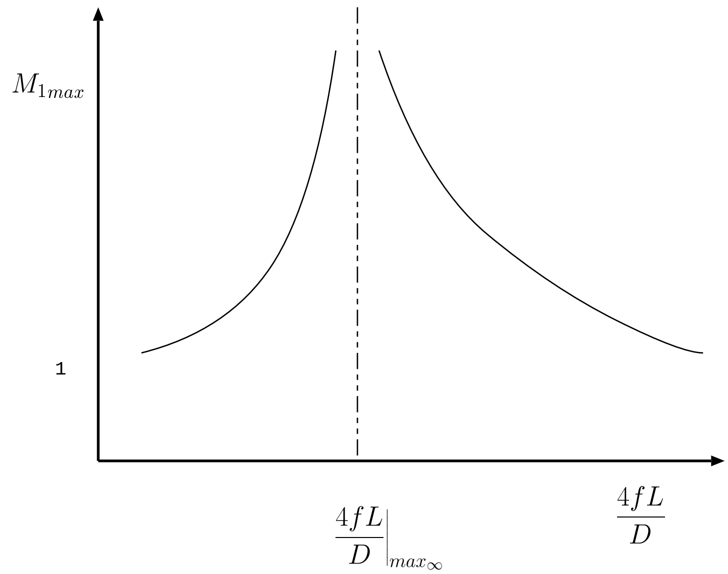


Fig. -A.17. The maximum entrance Mach number, M_1 to the tube as a function of $\frac{4fL}{D}$ supersonic branch

<u>SOLUTION</u>

The solution is obtained by an iterative process. The maximum $\left.\frac{4fL}{D}\right|_{max}$ for $k = 1.4$ is 0.821508116. Hence, $\frac{4fL}{D}$ exceed the maximum length $\frac{4fL}{D}$ for this entrance Mach number. The maximum for $M_1 = 8$ is $\frac{4fL}{D} = 0.76820$, thus the extra tube is $\Delta\left(\frac{4fL}{D}\right) = 0.9 - 0.76820 = 0.1318$. The left side is when the shock occurs at $\frac{4fL}{D} = 0.76820$ (flow is choked and no additional $\frac{4fL}{D}$). Hence, the value of left side is -0.1318. The right side is when the shock is at the entrance at which the extra $\frac{4fL}{D}$ is calculated for M_x and M_y is

$\mathbf{M_x}$	$\mathbf{M_y}$	$\frac{\mathbf{T_y}}{\mathbf{T_x}}$	$\frac{\rho_y}{\rho_x}$	$\frac{\mathbf{P_y}}{\mathbf{P_x}}$	$\frac{\mathbf{P_0}_y}{\mathbf{P_0}_x}$
8.0000	0.39289	13.3867	5.5652	74.5000	0.00849

With $(M_1)'$

$\mathbf{M}$	$\frac{\mathbf{4fL}}{\mathbf{D}}$	$\frac{\mathbf{P}}{\mathbf{P^*}}$	$\frac{\mathbf{P_0}}{\mathbf{P_0}^*}$	$\frac{\rho}{\rho^*}$	$\frac{\mathbf{U}}{\mathbf{U^*}}$	$\frac{\mathbf{T}}{\mathbf{T^*}}$
0.39289	2.4417	2.7461	1.6136	2.3591	0.42390	1.1641

The extra $\Delta\left(\frac{4fL}{D}\right)$ is $2.442 - 0.1318 = 2.3102$ Now the solution is somewhere between the negative of left side to the positive of the right side[17].

In a summary of the actions is done by the following algorithm:

(a) check if the $\frac{4fL}{D}$ exceeds the maximum $\frac{4fL}{D}_{max}$ for the supersonic flow. Accordingly continue.

(b) Guess $\frac{4fL}{D}_{up} = \frac{4fL}{D} - \left.\frac{4fL}{D}\right|_{max}$

(c) Calculate the Mach number corresponding to the current guess of $\frac{4fL}{D}_{up}$,

(d) Calculate the associate Mach number, M_x with the Mach number, M_y calculated previously,

(e) Calculate $\frac{4fL}{D}$ for supersonic branch for the M_x

(f) Calculate the "new and improved" $\frac{4fL}{D}_{up}$

(g) Compute the "new $\frac{4fL}{D}_{down} = \frac{4fL}{D} - \frac{4fL}{D}_{up}$

(h) Check the new and improved $\left.\frac{4fL}{D}\right|_{down}$ against the old one. If it is satisfactory stop or return to stage (b).

Shock location are:

$\mathbf{M_1}$	$\mathbf{M_2}$	$\left.\frac{\mathbf{4fL}}{\mathbf{D}}\right\|_{\mathbf{up}}$	$\left.\frac{\mathbf{4fL}}{\mathbf{D}}\right\|_{\mathbf{down}}$	$\mathbf{M_x}$	$\mathbf{M_y}$
8.0000	1.0000	0.57068	0.32932	1.6706	0.64830

The iteration summary is also shown below

[17]What if the right side is also negative? The flow is chocked and shock must occur in the nozzle before entering the tube. Or in a very long tube the whole flow will be subsonic.

$\mathbf{i}$	$\left.\frac{\mathbf{4fL}}{\mathbf{D}}\right\|_{\mathbf{up}}$	$\left.\frac{\mathbf{4fL}}{\mathbf{D}}\right\|_{\mathbf{down}}$	$\mathbf{M_x}$	$\mathbf{M_y}$	$\frac{\mathbf{4fL}}{\mathbf{D}}$
0	0.67426	0.22574	1.3838	0.74664	0.90000
1	0.62170	0.27830	1.5286	0.69119	0.90000
2	0.59506	0.30494	1.6021	0.66779	0.90000
3	0.58217	0.31783	1.6382	0.65728	0.90000
4	0.57605	0.32395	1.6554	0.65246	0.90000
5	0.57318	0.32682	1.6635	0.65023	0.90000
6	0.57184	0.32816	1.6673	0.64920	0.90000
7	0.57122	0.32878	1.6691	0.64872	0.90000
8	0.57093	0.32907	1.6699	0.64850	0.90000
9	0.57079	0.32921	1.6703	0.64839	0.90000
10	0.57073	0.32927	1.6705	0.64834	0.90000
11	0.57070	0.32930	1.6706	0.64832	0.90000
12	0.57069	0.32931	1.6706	0.64831	0.90000
13	0.57068	0.32932	1.6706	0.64831	0.90000
14	0.57068	0.32932	1.6706	0.64830	0.90000
15	0.57068	0.32932	1.6706	0.64830	0.90000
16	0.57068	0.32932	1.6706	0.64830	0.90000
17	0.57068	0.32932	1.6706	0.64830	0.90000

This procedure rapidly converted to the solution.

End Solution

A.10 The Practical Questions and Examples of Subsonic branch

The Fanno is applicable also when the flow isn't choke[18]. In this case, several questions appear for the subsonic branch. This is the area shown in Figure (A.8) in beginning for between points 0 and *a*. This kind of questions made of pair given information to find the conditions of the flow, as oppose to only one piece of information given in choked flow. There many combinations that can appear in this situation but there are several more physical and practical that will be discussed here.

[18] This questions were raised from many who didn't find any book that discuss these practical aspects and send questions to this author.

A.10.1 Subsonic Fanno Flow for Given $\frac{4fL}{D}$ and Pressure Ratio

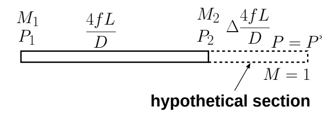


Fig. -A.18. Unchoked flow calculations showing the hypothetical "full" tube when choked

This pair of parameters is the most natural to examine because, in most cases, this information is the only information that is provided. For a given pipe $\left(\frac{4fL}{D}\right)$, neither the entrance Mach number nor the exit Mach number are given (sometimes the entrance Mach number is give see the next section). There is no exact analytical solution. There are two possible approaches to solve this problem: one, by building a representative function and find a root (or roots) of this representative function. Two, the problem can be solved by an iterative procedure. The first approach require using root finding method and either method of spline method or the half method found to be good. However, this author experience show that these methods in this case were found to be relatively slow. The Newton–Rapson method is much faster but not were found to be unstable (at lease in the way that was implemented by this author). The iterative method used to solve constructed on the properties of several physical quantities must be in a certain range. The first fact is that the pressure ratio P_2/P_1 is always between 0 and 1 (see Figure A.18). In the figure, a theoretical extra tube is added in such a length that cause the flow to choke (if it really was there). This length is always positive (at minimum is zero).

The procedure for the calculations is as the following:

1) Calculate the entrance Mach number, $M_1{}'$ assuming the $\frac{4fL}{D} = \left.\frac{4fL}{D}\right|_{max}{}'$ (chocked flow);

2) Calculate the minimum pressure ratio $(P_2/P_1)_{min}$ for $M_1{}'$ (look at table (A.1))

3) Check if the flow is choked:
   There are two possibilities to check it.

   a) Check if the given $\frac{4fL}{D}$ is smaller than $\frac{4fL}{D}$ obtained from the given P_1/P_2, or
   b) check if the $(P_2/P_1)_{min}$ is larger than (P_2/P_1),

   continue if the criteria is satisfied. Or if not satisfied abort this procedure and continue to calculation for choked flow.

4) Calculate the M_2 based on the $(P^*/P_2) = (P_1/P_2)$,

5) calculate $\Delta\frac{4fL}{D}$ based on M_2,

6) calculate the new (P_2/P_1), based on the new $f\left(\left(\frac{4fL}{D}\right)_1, \left(\frac{4fL}{D}\right)_2\right)$,
   (remember that $\Delta\frac{4fL}{D} = \left(\frac{4fL}{D}\right)_2$),

7) calculate the corresponding M_1 and M_2,

8) calculate the new and "improve" the $\Delta\frac{4fL}{D}$ by

$$\left(\Delta\frac{4fL}{D}\right)_{new} = \left(\Delta\frac{4fL}{D}\right)_{old} * \frac{\left(\frac{P_2}{P_1}\right)_{given}}{\left(\frac{P_2}{P_1}\right)_{old}} \tag{A.53}$$

Note, when the pressure ratios are matching also the $\Delta\frac{4fL}{D}$ will also match.

9) Calculate the "improved/new" M_2 based on the improve $\Delta\frac{4fL}{D}$

10) calculate the improved $\frac{4fL}{D}$ as $\frac{4fL}{D} = \left(\frac{4fL}{D}\right)_{given} + \Delta\left(\frac{4fL}{D}\right)_{new}$

11) calculate the improved M_1 based on the improved $\frac{4fL}{D}$.

12) Compare the abs $((P_2/P_1)_{new} - (P_2/P_1)_{old}$) and if not satisfied returned to stage (6) until the solution is obtained.

To demonstrate how this procedure is working consider a typical example of $\frac{4fL}{D} = 1.7$ and $P_2/P_1 = 0.5$. Using the above algorithm the results are exhibited in the following figure. Figure (A.19) demonstrates that the conversion occur at about 7-8

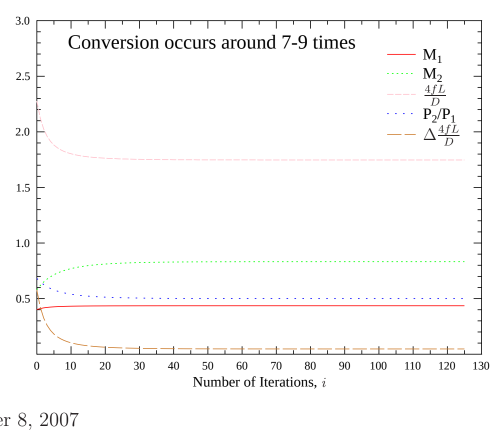


Fig. -A.19. The results of the algorithm showing the conversion rate for unchoked Fanno flow model with a given $\frac{4fL}{D}$ and pressure ratio.

iterations. With better first guess this conversion procedure will converts much faster (under construction).

A.10.2 Subsonic Fanno Flow for a Given M_1 and Pressure Ratio

This situation pose a simple mathematical problem while the physical situation occurs in cases where a specific flow rate is required with a given pressure ratio (range) (this problem was considered by some to be somewhat complicated). The specific flow rate can be converted to entrance Mach number and this simplifies the problem. Thus, the problem is reduced to find for given entrance Mach, M_1, and given pressure ratio calculate the flow parameters, like the exit Mach number, M_2. The procedure is based on the fact that the entrance star pressure ratio can be calculated using M_1. Thus, using the pressure ratio to calculate the star exit pressure ratio provide the exit Mach number, M_2. An example of such issue is the following example that combines also the "Naughty professor" problems.

Example A.4:
Calculate the exit Mach number for $P_2/P1 = 0.4$ and entrance Mach number $M_1 = 0.25$.

Solution

The star pressure can be obtained from a table or Potto-GDC as

$\mathbf{M}$	$\frac{\mathbf{4fL}}{\mathbf{D}}$	$\frac{\mathbf{P}}{\mathbf{P^*}}$	$\frac{\mathbf{P_0}}{\mathbf{P_0}^*}$	$\frac{\rho}{\rho^*}$	$\frac{\mathbf{U}}{\mathbf{U^*}}$	$\frac{\mathbf{T}}{\mathbf{T^*}}$
0.25000	8.4834	4.3546	2.4027	3.6742	0.27217	1.1852

And the star pressure ratio can be calculated at the exit as following

$$\frac{P_2}{P^*} = \frac{P_2}{P_1}\frac{P_1}{P^*} = 0.4 \times 4.3546 = 1.74184$$

And the corresponding exit Mach number for this pressure ratio reads

$\mathbf{M}$	$\frac{\mathbf{4fL}}{\mathbf{D}}$	$\frac{\mathbf{P}}{\mathbf{P^*}}$	$\frac{\mathbf{P_0}}{\mathbf{P_0}^*}$	$\frac{\rho}{\rho^*}$	$\frac{\mathbf{U}}{\mathbf{U^*}}$	$\frac{\mathbf{T}}{\mathbf{T^*}}$
0.60694	0.46408	1.7418	1.1801	1.5585	0.64165	1.1177

A bit show off the Potto–GDC can carry these calculations in one click as

$\mathbf{M_1}$	$\mathbf{M2}$	$\frac{\mathbf{4fL}}{\mathbf{D}}$	$\frac{\mathbf{P_2}}{\mathbf{P_1}}$
0.25000	0.60693	8.0193	0.40000

End Solution

While the above example show the most simple from of this question, in reality this question is more complicated. One common problem is situation that the diameter is not given but the flow rate and length and pressure (stagnation or static) with some combination of the temperature. The following example deal with one of such example.

Example A.5:

A tank filled with air at stagnation pressure, $2[Bar]$ should be connected to a pipe with a friction factor, $f = 0.005$, and and length of $5[m]$. The flow rate is (should be) 0.1$\left[\frac{kg}{sec}\right]$ and the static temperature at the entrance of the pipe was measured to be $27°C$. The pressure ratio P_2/P_1 should not fall below 0.9 ($P_2/P_1 > 0.9$). Calculate the exit Mach number, M_2, flow rate, and minimum pipe diameter. You can assume that $k = 1.4$.

Solution

The direct mathematical solution isn't possible and some kind of iteration procedure or root finding for a representative function. For the first part the "naughty professor" procedure cannot be used because $\dot{m}/A$ is not provided and the other hand $\frac{4fL}{D}$ is not provided (missing Diameter). One possible solution is to guess the entrance Mach and check whether and the mass flow rate with the "naughty professor" procedure are satisfied. For Fanno flow at for several Mach numbers the following is obtained

$\mathbf{M_1}$	M2	$\frac{\mathbf{4fL}}{\mathbf{D}}$	$\frac{\mathbf{P_2}}{\mathbf{P_1}}$	Diameter
0.10000	0.11109	13.3648	0.90000	0.00748
0.15000	0.16658	5.8260	0.90000	0.01716
0.20000	0.22202	3.1887	0.90000	0.03136

From the last table the diameter can be calculated for example for $M_1 = 0.2$ as

$$D = \frac{4fL}{\frac{4fL}{D}} = 4 \times 0.005 \times 5/3.1887 = 0.03136[m]$$

The same was done for all the other Mach number. Now the area can be calculated and therefor the $\dot{m}/A$ can be calculated. With this information the "naughty professor" is given and the entrance Mach number can be calculated. For example for $M_1 = 0.2$ one can obtain the following:

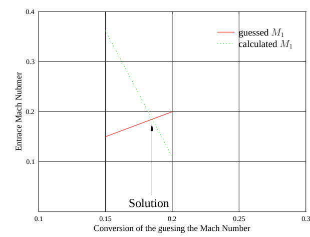
$$\dot{m}/A = 0.1/(\pi \times 0.03136^2/4) \sim 129.4666798$$

The same order as the above table it shown in "naughty professor" (isentropic table).

M	$\frac{\mathbf{T}}{\mathbf{T_0}}$	$\frac{\rho}{\rho_0}$	$\frac{\mathbf{A}}{\mathbf{A^\star}}$	$\frac{\mathbf{P}}{\mathbf{P_0}}$	$\frac{\mathbf{A\times P}}{\mathbf{A^*\times P_0}}$	$\frac{\mathbf{F}}{\mathbf{F^*}}$
1.5781	0.66752	0.36404	1.2329	0.24300	0.29960	0.56009
0.36221	0.97443	0.93730	1.7268	0.91334	1.5772	0.77785
0.10979	0.99760	0.99400	5.3092	0.99161	5.2647	2.2306

The first result are not reasonable and this process can continue until the satisfactory solution is achieved. Here an graphical approximation is shown.

From this exhibit it can be estimated that $M_1 = 0.18$. For this Mach number the following can be obtained



October 18, 2007

Fig. -A.20. Diagram for finding solution when the pressure ratio and entrance properties (T and P_0 are given

$\mathbf{M_1}$	$\mathbf{M2}$	$\frac{\mathbf{4fL}}{\mathbf{D}}$	$\frac{\mathbf{P_2}}{\mathbf{P_1}}$
0.18000	0.19985	3.9839	0.90000

Thus, the diameter can be obtained as $D \sim 0.0251[m]$
The flow rate is $\dot{m}/A \sim 202.1[kg/sec \times m^2]$

$\mathbf{M}$	$\frac{\mathbf{T}}{\mathbf{T_0}}$	$\frac{\rho}{\rho_0}$	$\frac{\mathbf{A}}{\mathbf{A^\star}}$	$\frac{\mathbf{P}}{\mathbf{P_0}}$	$\frac{\mathbf{A \times P}}{\mathbf{A^* \times P_0}}$	$\frac{\mathbf{F}}{\mathbf{F^*}}$
0.17109	0.99418	0.98551	3.4422	0.97978	3.3726	1.4628

The exact solution is between 0.17 to 0.18 if better accuracy is needed.

End Solution

A.11 The Approximation of the Fanno Flow by Isothermal Flow

The isothermal flow model has equations that theoreticians find easier to use and to compare to the Fanno flow model.

One must notice that the maximum temperature at the entrance is T_{01}. When the Mach number decreases the temperature approaches the stagnation temperature ($T \rightarrow T_0$). Hence, if one allows certain deviation of temperature, say about 1% that flow can be assumed to be isothermal. This tolerance requires that $(T_0 - T)/T_0 = 0.99$ which requires that enough for $M_1 < 0.15$ even for large $k = 1.67$. This requirement provides that somewhere (depend) in the vicinity of $\frac{4fL}{D} = 25$ the flow can be assumed isothermal. Hence the mass flow rate is a function of $\frac{4fL}{D}$ because M_1 changes. Looking at the table or Figure (A.2) or the results from Potto–GDC attached to this book shows that reduction of the mass flow is very rapid. As it can be seen for the Figure (A.21) the dominating parameter is $\frac{4fL}{D}$. The results are very similar for isothermal flow. The only difference is in small dimensionless friction, $\frac{4fL}{D}$.

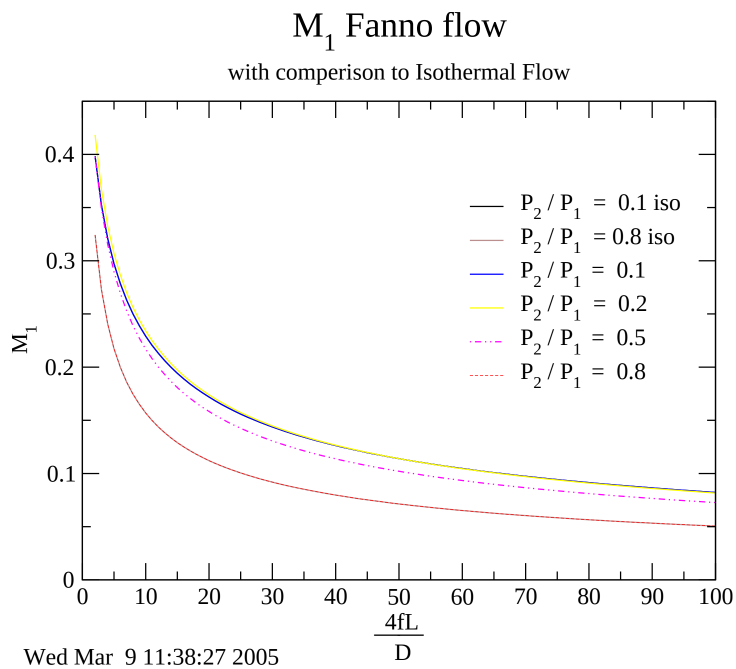


Fig. -A.21. The entrance Mach number as a function of dimensionless resistance and comparison with Isothermal Flow

A.12 More Examples of Fanno Flow

Example A.6:

To demonstrate the utility in Figure (A.21) *consider the following example. Find the mass flow rate for* $f = 0.05$, $L = 4[m]$, $D = 0.02[m]$ *and pressure ratio* $P_2/P_1 = 0.1, 0.3, 0.5, 0.8$. *The stagnation conditions at the entrance are* $300K$ *and* $3[bar]$ *air.*

<u>Solution</u>

First calculate the dimensionless resistance, $\frac{4fL}{D}$.

$$\frac{4fL}{D} = \frac{4 \times 0.05 \times 4}{0.02} = 40$$

From Figure (A.21) for $P_2/P_1 = 0.1$ $M_1 \approx 0.13$ etc.

or accurately by utilizing the program as in the following table.

$\mathbf{M_1}$	$\mathbf{M_2}$	$\frac{\mathbf{4fL}}{\mathbf{D}}$	$\left.\frac{\mathbf{4fL}}{\mathbf{D}}\right\|_{\mathbf{1}}$	$\left.\frac{\mathbf{4fL}}{\mathbf{D}}\right\|_{\mathbf{2}}$	$\frac{\mathbf{P_2}}{\mathbf{P_1}}$
0.12728	1.0000	40.0000	40.0000	0.0	0.11637
0.12420	0.40790	40.0000	42.1697	2.1697	0.30000
0.11392	0.22697	40.0000	50.7569	10.7569	0.50000
0.07975	0.09965	40.0000	107.42	67.4206	0.80000

Only for the pressure ratio of 0.1 the flow is choked.

$\mathbf{M}$	$\frac{\mathbf{T}}{\mathbf{T_0}}$	$\frac{\rho}{\rho_0}$	$\frac{\mathbf{A}}{\mathbf{A^\star}}$	$\frac{\mathbf{P}}{\mathbf{P_0}}$	$\frac{\mathbf{A\times P}}{\mathbf{A^*\times P_0}}$
0.12728	0.99677	0.99195	4.5910	0.98874	4.5393
0.12420	0.99692	0.99233	4.7027	0.98928	4.6523
0.11392	0.99741	0.99354	5.1196	0.99097	5.0733
0.07975	0.99873	0.99683	7.2842	0.99556	7.2519

Therefore, $T \approx T_0$ and is the same for the pressure. Hence, the mass rate is a function of the Mach number. The Mach number is indeed a function of the pressure ratio but mass flow rate is a function of pressure ratio only through Mach number.

The mass flow rate is

$$\dot{m} = PAM\sqrt{\frac{k}{RT}} = 300000 \times \frac{\pi \times 0.02^2}{4} \times 0.127 \times \sqrt{\frac{1.4}{287300}} \approx 0.48 \left(\frac{kg}{sec}\right)$$

and for the rest

$$\dot{m}\left(\frac{\mathbf{P_2}}{\mathbf{P_1}} = 0.3\right) \sim 0.48 \times \frac{0.1242}{0.1273} = 0.468 \left(\frac{kg}{sec}\right)$$

$$\dot{m}\left(\frac{\mathbf{P_2}}{\mathbf{P_1}} = 0.5\right) \sim 0.48 \times \frac{0.1139}{0.1273} = 0.43 \left(\frac{kg}{sec}\right)$$

$$\dot{m}\left(\frac{\mathbf{P_2}}{\mathbf{P_1}} = 0.8\right) \sim 0.48 \times \frac{0.07975}{0.1273} = 0.30 \left(\frac{kg}{sec}\right)$$

End Solution

A.13 The Table for Fanno Flow

Table -A.1. Fanno Flow Standard basic Table

$\mathbf{M}$	$\frac{\mathbf{4fL}}{\mathbf{D}}$	$\frac{\mathbf{P}}{\mathbf{P^*}}$	$\frac{\mathbf{P_0}}{\mathbf{P_0}^*}$	$\frac{\rho}{\rho^*}$	$\frac{\mathbf{U}}{\mathbf{U^*}}$	$\frac{\mathbf{T}}{\mathbf{T^*}}$
0.03	787.08	36.5116	19.3005	30.4318	0.03286	1.1998
0.04	440.35	27.3817	14.4815	22.8254	0.04381	1.1996

Table -A.1. Fanno Flow Standard basic Table (continue)

M	$\frac{4fL}{D}$	$\frac{P}{P^*}$	$\frac{P_0}{P_0{}^*}$	$\frac{\rho}{\rho^*}$	$\frac{U}{U^*}$	$\frac{T}{T^*}$
0.05	280.02	21.9034	11.5914	18.2620	0.05476	1.1994
0.06	193.03	18.2508	9.6659	15.2200	0.06570	1.1991
0.07	140.66	15.6416	8.2915	13.0474	0.07664	1.1988
0.08	106.72	13.6843	7.2616	11.4182	0.08758	1.1985
0.09	83.4961	12.1618	6.4613	10.1512	0.09851	1.1981
0.10	66.9216	10.9435	5.8218	9.1378	0.10944	1.1976
0.20	14.5333	5.4554	2.9635	4.5826	0.21822	1.1905
0.25	8.4834	4.3546	2.4027	3.6742	0.27217	1.1852
0.30	5.2993	3.6191	2.0351	3.0702	0.32572	1.1788
0.35	3.4525	3.0922	1.7780	2.6400	0.37879	1.1713
0.40	2.3085	2.6958	1.5901	2.3184	0.43133	1.1628
0.45	1.5664	2.3865	1.4487	2.0693	0.48326	1.1533
0.50	1.0691	2.1381	1.3398	1.8708	0.53452	1.1429
0.55	0.72805	1.9341	1.2549	1.7092	0.58506	1.1315
0.60	0.49082	1.7634	1.1882	1.5753	0.63481	1.1194
0.65	0.32459	1.6183	1.1356	1.4626	0.68374	1.1065
0.70	0.20814	1.4935	1.0944	1.3665	0.73179	1.0929
0.75	0.12728	1.3848	1.0624	1.2838	0.77894	1.0787
0.80	0.07229	1.2893	1.0382	1.2119	0.82514	1.0638
0.85	0.03633	1.2047	1.0207	1.1489	0.87037	1.0485
0.90	0.01451	1.1291	1.0089	1.0934	0.91460	1.0327
0.95	0.00328	1.061	1.002	1.044	0.95781	1.017
1.00	0.0	1.00000	1.000	1.000	1.00	1.000
2.00	0.30500	0.40825	1.688	0.61237	1.633	0.66667
3.00	0.52216	0.21822	4.235	0.50918	1.964	0.42857
4.00	0.63306	0.13363	10.72	0.46771	2.138	0.28571
5.00	0.69380	0.089443	25.00	0.44721	2.236	0.20000
6.00	0.72988	0.063758	53.18	0.43568	2.295	0.14634
7.00	0.75280	0.047619	$1.0E{+}2$	0.42857	2.333	0.11111
8.00	0.76819	0.036860	$1.9E{+}2$	0.42390	2.359	0.086957
9.00	0.77899	0.029348	$3.3E{+}2$	0.42066	2.377	0.069767
10.00	0.78683	0.023905	$5.4E{+}2$	0.41833	2.390	0.057143
20.00	0.81265	0.00609	$1.5E{+}4$	0.41079	2.434	0.014815
25.00	0.81582	0.00390	$4.6E{+}4$	0.40988	2.440	0.00952
30.00	0.81755	0.00271	$1.1E{+}5$	0.40938	2.443	0.00663
35.00	0.81860	0.00200	$2.5E{+}5$	0.40908	2.445	0.00488
40.00	0.81928	0.00153	$4.8E{+}5$	0.40889	2.446	0.00374
45.00	0.81975	0.00121	$8.6E{+}5$	0.40875	2.446	0.00296
50.00	0.82008	0.000979	$1.5E{+}6$	0.40866	2.447	0.00240

Table -A.1. Fanno Flow Standard basic Table (continue)

$\mathbf{M}$	$\frac{\mathbf{4fL}}{\mathbf{D}}$	$\frac{\mathbf{P}}{\mathbf{P^*}}$	$\frac{\mathbf{P_0}}{\mathbf{P_0}^*}$	$\frac{\rho}{\rho^*}$	$\frac{\mathbf{U}}{\mathbf{U^*}}$	$\frac{\mathbf{T}}{\mathbf{T^*}}$
55.00	0.82033	0.000809	$2.3E{+}6$	0.40859	2.447	0.00198
60.00	0.82052	0.000680	$3.6E{+}6$	0.40853	2.448	0.00166
65.00	0.82066	0.000579	$5.4E{+}6$	0.40849	2.448	0.00142
70.00	0.82078	0.000500	$7.8E{+}6$	0.40846	2.448	0.00122

A.14 Appendix – Reynolds Number Effects

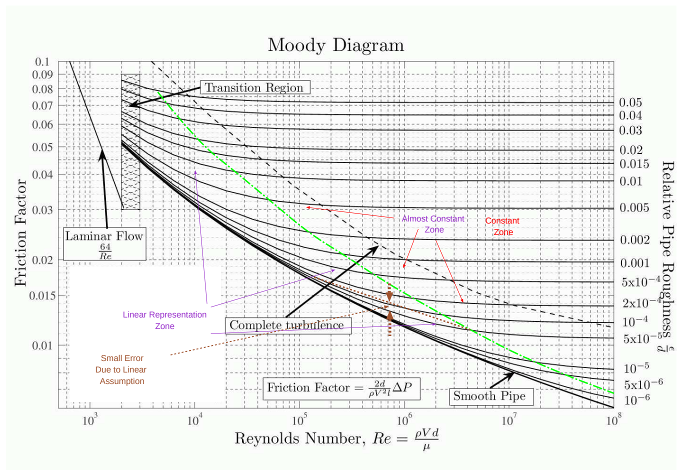


Fig. -A.22. "Moody" diagram on the name Moody who netscaped H. Rouse work to claim as his own. In this section the turbulent area is divided into 3 zones, constant, semi–constant, and linear After S Beck and R. Collins.

The friction factor in equation (A.25) was assumed constant. In Chapter **??** it was shown that the Reynolds number remains constant for ideal gas fluid. However, in Fanno flow the temperature does not remain constant hence, as it was discussed before, the Reynolds number is increasing. Thus, the friction decreases with the exception of the switch in the flow pattern (laminar to turbulent flow). For relatively large relative roughness larger $\epsilon/D > 0.004$ of 0.4% the friction factor is constant. For smother pipe $\epsilon/D < 0.001$ and Reynolds number between 10,000 to a million the friction factor vary between 0.007 to 0.003 with is about factor of two. Thus, the error of $\frac{4fL}{D}$ is limited by a factor of two (2). For this range, the friction factor can be estimated as a linear function of the $log_{10}(Re)$. The error in this assumption is probably small of the assumption that involve in fanno flow model construction. Hence,

$$f = A\,log_{10}(Re) + B \tag{A.54}$$

Where the constant A and B are function of the relative roughness. For most practical purposes the slop coefficient A can be further assumed constant. The slop coefficient $A = -0.998125$ Thus, to carry this calculation relationship between the viscosity and the temperature. If the viscosity expanded as Taylor or Maclaren series then

$$\frac{\mu}{\mu_1} = A_0 + \frac{A_1\,T}{T_0} + \cdots \tag{A.55}$$

Where μ_1 is the viscosity at the entrance temperature T_1.

Thus, Reynolds number is

$$Re = \frac{D\,\rho\,U}{A_0 + \frac{A_1\,T}{T_0} + \cdots} \tag{A.56}$$

Substituting equation (A.56) into equation (A.54) yield

$$f = A\,log_{10}\left(\frac{D\,\rho\,U}{A_0 + \frac{A_1\,T_2}{T_1} + \cdots}\right) + B \tag{A.57}$$

Left hand side of equation (A.25) is a function of the Mach number since it contains the temperature. If the temperature functionality will not vary similarly to the case of constant friction factor then the temperature can be expressed using equation (A.41).

$$\frac{4}{D}\left(A\,log_{10}\left(\frac{\overbrace{D\,\rho\,U}^{constant}}{A_0 + A_1\dfrac{1+\frac{k-1}{2}{M_1}^2}{1+\frac{k-1}{2}{M_2}^2} + \cdots}\right) + B\right) \tag{A.58}$$

Equation (A.58) is only estimate of the functionally however, this estimate is almost as good as the assumptions of Fanno flow. Equation fanno:eq:fld2 can be improved by using equation (A.58)

$$\frac{4\,L_{max}}{D}\left(A\,log_{10}\left(\frac{\overbrace{D\,\rho\,U}^{constant}}{A_0 + A_1\dfrac{1+\frac{k-1}{2}M^2}{1+\frac{k-1}{2}}}\right) + B\right) \sim \frac{1}{k}\frac{1-M^2}{M^2} + \frac{k+1}{2k}\ln\frac{\frac{k+1}{2}M^2}{1+\frac{k-1}{2}M^2} \tag{A.59}$$

In the most complicate case where the flow pattern is change from laminar flow to turbulent flow the whole Fanno flow model is questionable and will produce poor results.

In summary, in the literature there are three approaches to this issue of non constant friction factor. The friction potential is recommended by a researcher in Germany and it is complicated. The second method substituting this physical approach with numerical iteration. In the numerical iteration method, the expression of the various relationships are inserted into governing differential equations. The numerical methods does not allow flexibility and is very complicated. The methods described here can be expended (if really really needed) and it will be done in very few iteration as it was shown in the Isothermal Chapter.

APPENDIX B

What The Establishment's Scientists Say

What a **Chutzpah***? to say samething like that!*

anonymous

In this section exhibits the establishment "experts" reaction the position that the "common" pQ^2 diagram is improper. Their comments are responses to the author's paper: "The mathematical theory of the pQ^2 diagram" (similar to Chapter 7)[1]. The paper was submitted to Journal of Manufacturing Science and Engineering.

This part is for the Associate Technical Editor Dr. R. E. Smelser.

I am sure that you are proud of the referees that you have chosen and that you do not have any objection whatsoever with publishing this information. Please send a copy of this appendix to the referees. I will be glad to hear from them.

This concludes comments to the Editor.

I believe that you, the reader should judge if the mathematical theory of the pQ^2 diagram is correct or whether the "experts" position is reasonable. For the reader unfamiliar with the journal review process, the associate editor sent the paper to "readers" (referees) which are anonymous to the authors. They comment on the paper and according to these experts the paper acceptance is determined. I have chosen the unusual step to publish their comments because I believe that other motivations are involved in their responses. Coupled with the response to the publication of a summary of this

[1]The exact paper can be obtain free of charge from Minnesota Suppercomputing Institute, http://www2.msi.umn.edu/publications.html report number 99/40 "The mathematical theory of the pQ^2 diagram" or by writing to the Supercomputing Institute, University of Minnesota, 1200 Washington Avenue South, Minneapolis, MN 55415-1227

paper in the Die Casting Engineer, bring me to think that the best way to remove the information blockage is to open it to the public.

Here, the referees can react to this rebuttal and stay anonymous via correspondence with the associated editor. If the referee/s choose to respond to the rebuttal, their comments will appear in the future additions. I will help them as much as I can to show their opinions. I am sure that they are proud of their criticism and are standing behind it 100%. Furthermore, I am absolutely, positively sure that they are so proud of their criticism they glad that it appears in publication such as this book.

B.1 Summary of Referee positions

The critics attack the article in three different ways. All the referees try to deny publication based on grammar!! The first referee didn't show any English mistakes (though he alleged that he did). The second referee had some hand written notes on the preprint (two different hand writing?) but it is not the grammar but the content of the article (the fact that the "common" pQ^2 diagram is wrong) is the problem.

Here is an original segment from the submitted paper:

> The design process is considered an art for the 8–billion–dollar die casting industry. The pQ^2 diagram is the most common calculation, if any that all, are used by most die casting engineers. The importance of this diagram can be demonstrated by the fact that tens of millions of dollars have been invested by NADCA, NSF, and other major institutes here and abroad in pQ^2 diagram research.

In order to correct "grammar", the referee change to:

> The pQ^2 diagram is the most common calculation used by die casting engineers to determine the relationship between the die casting machine and gating design parameters, and the resulting metal flow rate.

It seems, the referee would not like some facts to be written/known.

Summary of the referees positions:

Referee 1 Well, the paper was published before (NADCA die casting engineer) and the errors in the "common" pQ^2 are only in extreme cases. Furthermore, it actually supports the "common" model.

Referee 2 Very angrily!! How dare the authors say that the "common" model is wrong. When in fact, according to him, it is very useful.

Referee 3 The bizzarro approach! Changed the meaning of what the authors wrote (see the "ovaled boxed" comment for example). This produced a new type logic which is almost absurd. Namely, the discharge coefficient, C_D, is constant for a runner or can only vary with time. The third possibility, which is the topic of the paper, the fact that C_D cannot be assigned a runner system but have to calculated for every set of runner and die casting machine can not exist possibility, and therefore the whole paper is irrelevant.

Genick Bar–Meir's answer:

Let me say what a smart man once said before: I don't need 2000 scientists to tell me that I am wrong. What I need is one scientist to show what is wrong in my theory.

Please read my rebuttal to the points the referees made. The referees version are kept as close as possible to the original. I put some corrections in a square bracket [] to clarify the referees point.

Referee comments appear in roman font like this sentence,

`and rebuttals appear in a courier font as this sentence.`

B.2 Referee 1 (from hand written notes)

1. Some awkward grammar – See highlighted portions

   `Where?`

2. Similarity of the submitted manuscript to the attached Die Casting Engineer Trade journal article (May/June 1998) is Striking.

   `The article in Die Casting Engineer is a summary of the present article. It is mentioned there that it is a summary of the present article. There is nothing secret about it. This article points out that the ''common'' model is totally wrong. This is of central importance to die casting engineers. The publication of this information cannot be delayed until the review process is finished.`

3. It is not clear to the reader why the "constant pressure" and "constant power" situations were specifically chosen to demonstrate the author's point. Which situation is most like that found [likely found] in a die casting machine? Does the "constant pressure" correspond closely to older style machines when intensifyer [intensifier] bottle pressure was applied to the injection system unthrottled? Does the "constant power" situations assume a newer machines, such as Buher Sc, that generates the pressure required to achieve a desired gate velocity? Some explanation of the logic of selecting these two situations would be helpful in the manuscript.

   `As was stated in the article, these situations were chosen because they are building blocks but more importantly to demonstrate that the ''common'' model is totally wrong! If it is wrong for two basic cases it should be absolutely wrong in` `any combinations of the two cases. Nevertheless, an additional explanation is given in Chapter 7.`

4. The author's approach is useful? Gives perspective to a commonly used process engineering method (pQ^2) in die casting. Some of the runner lengths chosen (1 meter) might be consider exceptional in die casting – yet the author uses this to show how much in error an "average" value for C_D be. The author might also note that the North American Die Casting Association and many practitioners use a A_3/A_2 ratio of $\approx .65$ as a design target for gating. The author" analysis reinforces this value as a good target, and that straying far from it may results in poor design part filling problems (Fig. 5)

   The reviewer refers to several points which are important to address. All the four sizes show large errors (we do not need to take 1[m] to demonstrate that). The one size, the referee referred to as exceptional (1 meter), is not the actual length but the represented length (read the article again). Poor design can be represented by a large length. This situation can be found throughout the die casting industry due to the ''common'' model which does not consider runner design. My office is full with runner designs with represent 1 meter length such as one which got NADCA's design award[2].

   In regards to the area ratio, please compare with the other referee who claim A_2/A_2 = 0.8 - 0.95. I am not sure which of you really represent NADCA's position (I didn't find any of NADCA's publication in regards to this point). I do not agree with both referees. This value has to be calculated and cannot be speculated as the referees asserted. Please find an explanation to this point in the paper or in even better in Chapter 7.

B.3 Referee 2

There are several major concerns I have about this paper. The [most] major one [of these] is that [it] is unclear what the paper is attempting to accomplish. Is the paper trying to suggest a new way of designing the rigging for a die casting, or is it trying to add an improvement to the conventional pQ^2 solution, or is it trying to somehow suggest a 'mathematical basis for the pQ^2 diagram'?

The paper shows that 1) the ''conventional pQ^2 solution'' is totally wrong, 2) the mathematical analytical solution for the pQ^2 provides an excellent tool for studying the effect of various parameters.

[2] to the best of my understanding

The other major concern is the poor organization of the ideas which the authors [are] trying to present. For instance, it is unclear how specific results presented in the results section where obtained ([for instance] how were the results in Figures 5 and 6 calculated?).

`I do not understand how the organization of the paper relates to the fact that the referee does not understand how Fig 5 and 6 were calculated. The organization of the paper does not have anything do with his understanding the concepts presented. In regard to the understanding of how Figure 5 and 6 were obtained, the referee should referred to an elementary fluid mechanics text book and combined it with the explanation presented in the paper.`

Several specific comments are written on the manuscript itself; most of these were areas where the reviewer was unclear on what the authors meant or areas where further discussion was necessary. One issue that is particularly irksome is the authors tendency in sections 1 and 2 to wonder [wander] off with "editorials" and other unsupported comments which have no place in a technical article.

`Please show me even one unsupported comment!!`

Other comments/concerns include-

- what does the title have to do with the paper? The paper does not define what a pQ2 diagram is and the results don't really tie in with the use of such diagrams.

  The paper presents the exact analytical solution for the pQ^2 diagram. The results tie in very well with the correct pQ^2 diagram. Unfortunately, the ''common'' model is incorrect and so the results cannot be tied in with it.

- p.4 The relationship $Q \propto \sqrt{P}$ is a result of the application of Bernoulli's equation system like that shown in Fig 1. What is the rational or basis behind equation 1; e.g. $Q \propto (1 - P)^n$ with n =1, 1/2, and 1/4?

  `Here I must thank the referee for his comment! If the referee had serious problem understanding this point, I probably should have considered adding a discussion about this point, such as in Chapter 7.`

- p.5 The relationship between equation 1(a) to 1(c) and a die casting machine as "poor", "common", and "excellent" performance is not clear and needs to be developed, or at least defined.

  `see previous comment`

- It is well known that C_D for a die casting machine and die is not a constant. In fact it is common practice to experimentally determine C_D for use on dies with 'similar' gating layouts in the future. But because most dies have numerous gates branching off of numerous runners, to determine all of the friction factors as a function of Reynolds number would be quite difficult and virtually untractable for design purposes. Generally die casting engineers find conventional pQ2 approach works quite well for design purposes.

This ''several points' comment give me the opportunity to discusses the following points:

- ★ I would kindly ask the referee, to please provide the names of any companies whom ''experimentally determine C_D.'' Perhaps they do it down under (Australia) where the ''regular'' physics laws do not apply (please, forgive me about being the cynical about this point. I cannot react to this any other way.). Please, show me a company that uses the ''common'' pQ^2 diagram and it works.
- ★ Due to the computer revolution, today it is possible to do the calculations of the C_D for a specific design with a specific flow rate (die casting machine). In fact, this is exactly what this paper all about. Moreover, today there is a program that already does these kind of calculations, called DiePerfect™.
- ★ Here the referee introduce a new idea of the ''family'' -- the improved constant C_D. In essence, the idea of ''family'' is improve constant C_D in which one assigned value to a specific group of runners. Since this idea violate the basic physics laws and the produces the opposite to realty trends it must be abandoned. Actually, the idea of ''family'' is rather bizarre, because a change in the design can lead to a significant change in the value of C_D. Furthermore, the ''family'' concept can lead to a poor design (read about this in the section ''poor design effects'' of this book). How one can decided which design is part of what ''family''? Even if there were no mistakes, the author's method (calculating the C_D) is of course cheaper and faster than the referee's suggestion about ''family'' of runner design. In summary, this idea a very bad idea.
- ★ What is C_D =constant? The referee refers to the case where C_D is constant for specific runner design but which is not the case in reality. The C_D does not depend only on the runner, but on the combination of the runner system with the die casting machine via the Re number. Thus, a specific runner design cannot have C_D ''assigned'' to it.

The C_D has to be calculated for any combination runner system with die casting machine.

$\star$ I would like to find any case where the ''common'' pQ^2 diagram does work. Please read the proofs in Chapter 7 showing why it cannot work.

- Discussion and results A great deal of discussion focuses on the regions where A_3/A_2 0.1; yet in typical die casting dies A_3/A_2 0.8 to 0.95.

  Please read the comments to the previous referee

In conclusion, it's just a plain sloppy piece of work

I hope that referee does not mind that I will use it as the chapter quote.

(the Authors even have one of the references to their own publications sited incorrectly!).

Perhaps, the referee should learn that magazines change names and, that the name appears in the reference is the magazine name at the time of writing the paper.

B.4 Referee 3

The following comments are not arranged in any particular order.

General: The text has a number of errors in grammar, usage and spelling that need to be addressed before publication.

p 6 1^{st} paragraph - The firsts sentence says that the flow rate is a function of temperature, yet the rest of the paragraph says that it isn't.

The rest of the paragraph say the flow rate is a weak function of the temperature and that it explains why. I hope that everyone agrees with me that it is common to state a common assumption and explain why in that particular case it is not important. I wish that more people would do just that. First, it would eliminate many mistakes that are synonymous with research in die casting, because it forces the ''smart'' researchers to check the major assumptions they make. Second, it makes clear to the reader why the assumption was made.

p 6 - after Eq 2 - Should indicate immediately that the subscript[s] refer to the sections in Figure 1.

I will consider this, Yet, I am not sure this is a good idea.

p 6 - after equation 2 - There is a major assumption made here that should not pass without some comment[s][3] "Assuming steady state " - This assumption goes

[3] Is the referee looking for one or several explanations?

to the heart of this approach to the filling calculation and establishes the bounds of its applicability. The authors should discuss this point.

`Well, I totally disagree with the referee on this point. The major question in die casting is how to ensure the right range of filling time and gate velocity. This paper's main concern is how to calculate the` C_D `and determine if the` C_D `be ''assigned'' to a specific runner. The unsteady state is only a side effect and has very limited importance due to AESS. Of course the flow is not continuous/steady state and is affected by many parameters such as the piston weight, etc, all of which are related to the transition point and not to the pQ`$^2$ `diagram per se. The unsteady state exists only in the initial and final stages of the injection. As a general rule, having a well designed pQ`$^2$ `diagram will produce a significant improvement in the process design. It should be noted that a full paper discussing the unsteady state is being prepared for publication at the present time.`

In general the organization of the paper is somewhat weak - the introduction especially does not very well set the technical context for the pQ2 method and show how the present work fits into it.

`The present work does not fit into past work! It shows that the past work is wrong, and builds the mathematical theory behind the pQ`^{2} `diagram.`

The last paragraph of the intro is confused [confusing]. The idea introduced in the last sentence on page 2 is that the CD should vary somehow during the calculation, and subsequently variation with Reynolds number is discussed, but the intervening material about geometry effects is inconsistent with a discussion of things that can vary during the calculation. The last two sentences do not fit together well either - "the assumption of constant CD is not valid" - okay, but is that what you are going to talk about, or are you going to talk about "particularly the effects of the gate area"?

`Firstly,` C_D `should not vary during the calculations it is a constant for a specific set of runner system and die casting machine. Secondly, once any parameter is changed, such as gate area,` C_D `has to be recalculated. Now the referee's statement` C_D `should vary, isn't right and therefore some of the following discussion is wrong.`
`Now about the fitting question. What do referee means by ''fit together?'' Do the paper has to be composed in a rhyming verse? Anyhow, geometrical effects are part of Reynolds number (review fluid mechanics). Hence, the effects of the gate area shows that` C_D `varies as well and has to be recalculated. So what is inconsistent? How do these sentences not fit together?`

On p 8, after Eq 10 - I think that it would be a good idea to indicate immediately

that these equations are plotted in Figure 3, rather than waiting to the next section before referring to Fig 3.
Also, making the Oz-axis on this graph logarithmic would help greatly in showing the differences in the three pump characteristics.

`Mentioning the figure could be good idea but I don't agree with you about the log scale, I do not see any benefits.`

On p. 10 after Eq 11 - The solution of Eq 11 requires full information on the die casting machine - According to this model, the machine characterized by Pmax, Qmax and the exponent in Eq 1. The wording of this sentence, however, might be indicating that there is some information to be had on the machine other that these three parameters. I do not think that that is what the authors intend, but this is confusing.

`This is exactly what the authors intended. The model does not confined to a specific exponent or function, but rather gives limiting cases. Every die casting machine can vary between the two extreme functions, as discussed in the paper. Hence, more information is needed for each individual die casting machine.`

p 12 - I tend to disagree with the premises of the discussion following Eq 12. I think that Qmax depends more strongly on the machine size than does Pmax. In general, P max is the intensification pressure that one wants to achieve during solidification, and this should not change much with the machine size, whereas the clamping force, the product of this pressure and platten are, goes up. On the other hand, when one has larger area to make larger casting, one wants to increase the volumetric flow rate of metal so that flow rate of metal so that fill times will not go up with the machine size. Commonly, the shot sleeve is larger, while the maximum piston velocity does not change much.

Here the referee is confusing so many different concepts that it will take a while to explain it properly. Please find here a attempt to explain it briefly. The intensification pressure has nothing to do with the pQ^2 diagram. The pQ^2 does not have much to do with the solidification process. It is designed not to have much with the solidification. The intensification pressure is much larger than P_{max}. I give up!! It would take a long discussion to teach you the fundamentals of the pQ^2 diagram and the die casting process. You confuse so many things that it impossible to unravel it all for you in a short paragraph. Please read Chapter 7 or even better read the whole book.

Also, following Eq 13, the authors should indicate what they mean by "middle range" of the Oz numbers. It is not clear from Fig 3 how close one needs to get to Oz=0 for the three curves to converge again.

`The mathematical equations are given in the paper. They are very simple that you can use hand calculator to find how much close you` `need to go to` $Oz = 0$ `for your choice of error. A discussion on such issue is below the level from an academic paper.`

.

Besides being illustrative of the results, part of the value of an example calculation comes from it making possible duplication of the results elsewhere. In order to support this, the authors need to include the relationships that used for CD in these calculations.

`The literature is full of such information. If the referee opens any basic fluid mechanics text book then he can find information about it.`

The discussion on p 14 of Fig 5 needs a little more consideration. There is a maximum in this curve, but the author's criterion of being on the "right hand branch" is said to be shorter fill time, which is not a criterion for choosing a location on this curve at all. The fill time is monotone decreasing with increasing A3 at constant A2, since the flow is the product of Vmax and A3. According to this criterion, no calculation is needed - the preferred configuration is no gate whatsoever. Clearly, choosing an operating point requires introduction of other criteria, including those that the authors mention in the intro. And the end of the page 14 discussion that the smaller filling time from using a large gate (or a smaller runner!!??) will lead to a smaller machine just does not follow at all. The machine size is determined by the part size and the required intensification pressure, not by any of this.

`Once again the referee is confusing many issues; let me interpret again what is the` pQ^2 `diagram is all about. The` pQ^2 `diagram is for having an operational point at the right gate velocity and the right filling time. For any given` A_2`, there are two possible solutions on the right hand side and one on the left hand side with the same gate velocity. However, the right hand side has smaller filling time. And again, the referee confusing another issue. Like in many engineering situations, we have here a situation in which more than one criterion is needed. The clamping force is one of the criteria that determines what machine should be chosen. The other parameter is the` $pQ^2$ `diagram.`

It seems that they authors have obscured some elementary results by doing their calculations.[4]

> For example, the last sentence of the middle paragraph on p 15 illustrates that as CD reaches its limiting value of 1, the discharge velocity reaches its maximum. This is not something we should be publishing in 1998.

C_D`? There is no mention of the alleged fact of ''`C_D `reaches its limiting value of 1.'' There is no discussion in the whole article`

[4] If it is so elementary how can it be obscured.
I have broken–out this paragraph for purposes of illustration.

`about ``C_D reaching its maximum ($C_D = 1$)''. Perhaps the referee was mistakenly commenting on different articles (NADCA's book or an other die casting book) which he has confused with this article.`

Regarding the concluding paragraph on p 15:

1. The use of the word "constant" is not consistent throughout this paper. Do they mean constant across geometry or constant across Reynolds number, or both.

   `To the readers: The referee means across geometry as different geometry and across Reynolds number as different Re number`[5]`. I really do not understand the difference between the two cases. Aren't actually these cases the same? A change in geometry leads to a change in Reynolds number number. Anyhow, the referee did not consider a completely different possibility. Constant C_D means that C_D is assigned to a specific runner system, or like the ``common'' model in which all the runners in the world have the same value.`

2. Assuming that they mean constant across geometry, then obviously, using a fixed value for all runner/gate systems will sometimes lead to large errors. They did not need to do a lot calculation to determine this.

   `And yet this method is the most used method in the industry(some even will say the exclusive method).`

3. Conversely, if they mean constant across Reynolds number, i.e. CD can vary through the run as the velocity varies, then they have not made their case very well. Since they have assumed steady state and the P3 does not enter into the calculation, then the only reason that mention for the velocity to vary during the fill would be because Kf varies as a function of the fill fraction. They have not developed this argument sufficiently.

   `Let me stress again the main point of the article. $C_D$ varies for different runners and/or die casting machines. It is postulated that the velocity does not vary during run. A discussion about $P_3$ is an entirely different issue related to the good venting design for which $P_3$ remains constant.`

4. If the examples given in the paper do not represent the characteristics of a typical die casting machine, why to present them at all? Why are the "more detailed calculations" not presented, instead of the trivial results that are shown?

[5] if the interpretation is not correct I would like to learn what it really mean.

These examples demonstrate that the ''common'' method is erroneous and that the ''authors''' method should be adopted or other methods based on scientific principles. I believe that this is a very good reason.

APPENDIX C

My Relationship with Die Casting Establishment

I cannot believe the situation that I am in. The hostility I am receiving from the establishment is unbelievable, as individual who has spent the last 12 years in research to improve the die casting. At first I was expecting to receive a welcome to the club. Later when my illusions disappeared, I realized that it revolves around money along with avoiding embarrassment to the establishment due to exposing of the truths and the errors the establishment has sponsored. I believe that the establishment does not want people to know that they had invested in research which produces erroneous models and continues to do so, even though they know these research works/models are scientifically rubbish. They don't want people to know about their misuse of money.

When I started my research, I naturally called what was then SDCE. My calls were never returned. A short time later SDCE developed into what is now called NADCA. I had hoped that this new creation would prove better. Approximately two years ago I wrote a letter to Steve Udvardy, director of research and education for NADCA (a letter I never submitted). Now I have decided that it is time to send the letter and to make it open to the public. I have a long correspondence with Paul Bralower, former director of communication for NADCA, which describes my battle to publish important information. An open letter to Mr. Baran, Director of Marketing for NADCA, is also attached. Please read these letters. They reveal a lot of information about many aspects of NADCA's operations. I have submitted five (5) articles to this conference (20th in Cleveland) and only one was accepted (only 20% acceptance compared to $\sim$ 70% to any body else). Read about it here. During my battle to "insert" science in die casting, many curious things have taken place and I wonder: are they coincidental? Read about these and please let me know what you think.

Open Letter to Mr. Udvardy

Steve Udvardy NADCA,
9701 West Higgins Road, Suite 880
Rosemont, IL 60018-4721

January 26, 1998

Subject: Questionable ethics

Dear Mr. Udvardy:

I am writing to express my concerns about possible improprieties in the way that NADCA awards research grants. As a NADCA member, I believe that these possible improprieties could result in making the die casting industry less competitive than the plastics and other related industries. If you want to enhance the competitiveness of the die casting industry, you ought to support die casting industry ethics and answer the questions that are raised herein.

Many of the research awards raise serious questions and concerns about the ethics of the process and cast very serious shadows on the integrity those involved in the process. In the following paragraphs I will spell out some of the things I have found. I suggest to you and all those concerned about the die casting industry that you/they should help to clarify these questions, and eliminate other problems if they exist in order to increase the die casting industry's profits and competitiveness with other industries. I also wonder why NADCA demonstrates no desire to participate in the important achievements I have made.

On September 26, 1996, I informed NADCA that Garber's model on the critical slow plunger velocity is unfounded, and, therefore so, is all the other research based on Garber's model (done by Dr. Brevick from Ohio State University). To my great surprise I learned from the March/April 1997 issue of Die Casting Engineer that NADCA has once again awarded Dr. Brevick with a grant to continue his research in this area. Also, a year after you stated that a report on the results from Brevick's could be obtained from NADCA, no one that I know of has been able to find or obtain this report. I and many others have tried to get this report, but in vain. It leaves me wondering whether someone does not want others to know about this research. I will pay $50 to the first person who will furnish me with this report. I also learned (in NADCA's December 22, 1997 publication) that once more NADCA awarded Dr. Brevick with another grant to do research on this same topic for another budget year (1998). Are Dr. Brevick's results really that impressive? Has he changed his model? What is the current model? Why have we not heard about it?

I also learned in the same issue of Die Casting Engineer that Dr. Brevick and his colleagues have been awarded another grant on top of the others to do research on the topic entitled "Development and Evaluation of the Sensor System." In the September/October 1997 issue, we learned that Mr. Gary Pribyl, chairman of the NADCA Process Technology Task Group, is part of the research team. This Mr. Pribyl is the chairman of the very committee which funded the research. Of course, I am sure, this could not be. I just would like to hear your explanation. Is it legitimate/ethical to have a man on the committee awarding the chairman a grant?

Working on the same research project with this Mr. Pribyl was Dr. Brevick who also received a grant mentioned above. Is there a connection between the fact that Gary Pribyl cooperated with Dr. B. Brevick on the sensor project and you deciding to renew Dr. Brevick's grant on the critical slow plunger velocity project? I would like to learn what the reasoning for continuing to fund Dr. Brevick after you had learned that his research was problematic.

Additionally, I learned that Mr. Steve Udvardy was given a large amount of money to study distance communications. I am sure that Mr. Udvardy can enhance NADCA's ability in distance learning and that this is why he was awarded this grant. I am also sure that Mr. Udvardy has all the credentials needed for such research. One can only wonder why his presentation was not added to the NADCA proceedings. One may also wonder why there is a need to do such research when so much research has already been done in this area by the world's foremost educational experts. Maybe it is because distance communication works differently for NADCA. Is there a connection between Mr. Steve Udvardy being awarded this grant and his holding a position as NADCA's research director? I would like to learn the reasons you vouchsafe this money to Mr. Udvardy! I also would like to know if Mr. Udvardy's duties as director of education include knowledge and research in this area. If so, why is there a need to pay Mr. Udvardy additional monies to do the work that he was hired for in the first place?

We were informed by Mr. Walkington on the behalf of NADCA in the Nov–Dec 1996 issue that around March or April 1997, we would have the software on the critical slow plunger velocity. Is there a connection between this software's apparent delayed appearance and the fact that the research in Ohio has produced totally incorrect and off–base results? I am sure that there are reasons preventing NADCA from completing and publishing this software; I would just like to know what they are. I am also sure that the date this article came out (Nov/Dec 1996) was only coincidentally immediately after I sent you my paper and proposal on the shot sleeve (September 1996). What do you think?

Likewise, I learned that Mr. Walkington, one of the governors of NADCA, also received a grant. Is there a connection between this grant being awarded to Mr. Walkington and his position? What about the connection between his receiving the grant and his former position as the director of NADCA research? I am sure that grant was awarded based on merit only. However, I have serious concerns about his research. I am sure that these concerns are unfounded, but I would like to know what Mr. Walkington's credentials are in this area of research.

The three most important areas in die casting are the critical slow plunger velocity, the pQ^2 diagram, and the runner system design. The research sponsored by NADCA on the critical slow plunger velocity is absolutely unfounded because it violates the basic physics laws. The implementation of the pQ^2 diagram is also absolutely unsound because again, it violates the basic physics laws. One of the absurdities of the previous model is the idea that plunger diameter has to decrease in order to increase the gate velocity. This conclusion (of the previous model)

violates several physics laws. As a direct consequence, the design of the runner system (as published in NADCA literature) is, at best, extremely wasteful.
As you also know, NADCA, NSF, the Department of Energy, and others sponsoring research in these areas exceed the tens of millions, and yet produce erroneous results. I am the one who discovered the correct procedure in both areas. It has been my continuous attempt to make NADCA part of these achievements. Yet, you still have not responded to my repeated requests for a grant. Is there a reason that it has taken you $1\frac{1}{2}$ years to give me a negative answer? Is there a connection between any of the above information and how long it has taken you?
Please see the impressive partial–list of the things that I have achieved. I am the one who found Garber's model to be totally and absolutely wrong. I am also the one responsible for finding the pQ^2 diagram implementation to be wrong. I am the one who is responsible for finding the correct pQ^2 diagram implementation. I am the one who developed the critical area concept. I am the one who developed the economical runner design concept. In my years of research in the area of die casting I have not come across any research that was sponsored by NADCA which was correct and/or which produced useful results!! Is there any correlation between the fact that all the important discoveries (that I am aware of) have been discovered not in–but outside of NADCA? I would like to hear about anything in my area of expertise supported by NADCA which is useful and correct? Is there a connection between the foregoing issues and the fact that so many of the die casting engineers I have met do not believe in science?
More recently, I have learned that your secretary/assistant, Tricia Margel, has now been awarded one of your grants and is doing research on pollution. I am sure the grant was given based on qualification and merit only. I would like to know what Ms. Margel's credentials in the pollution research area are? Has she done any research on pollution before? If she has done research in that area, where was it published? Why wasn't her research work published? If it was published, where can I obtain a copy of the research? Is this topic part of Ms. Margel's duties at her job? If so, isn't this a double payment? Or perhaps, was this an extra separated payment? Where can I obtain the financial report on how the money was spent?
Together we must promote die casting knowledge. I am doing my utmost to increase the competitiveness of the die casting industry with our arch rivals: the plastics industry, the composite material industry, and other industries. I am calling on everyone to join me to advance the knowledge of the die casting process.

Thank you for your consideration.
Sincerely,
Genick Bar–Meir, Ph.D.

cc: NADCA Board of Governors
NADCA members
Anyone who care about die casting industry

Correspondence with Paul Bralower

Paul Bralower is the former director of communications at NADCA. I have tried to publish articles about critical show shot sleeve and the pQ^2 diagram through NADCA magazine. Here is an example of my battle to publish the article regarding pQ^2 . You judge whether NADCA has been enthusiastic about publishing this kind of information. Even after Mr. Bralower said that he would publish it I had to continue my struggle.

He agreed to publish the article but ···

At first I sent a letter to Mr. Bralower (Aug 21, 1997):
Paul M. Bralower
NADCA, Editor
9701 West Higgins Road, Suite 880
Rosemont, IL 60018-4721

Dear Mr. Bralower:

Please find enclosed two (2) copies of the paper "The mathematical theory of the pQ^2 diagram" submitted by myself for your review. This paper is intended to be considered for publication in *Die Casting Engineer*.
For your convenience I include a disk DOS format with Microsoft WORD for window format (pq2.wid) of the paper, postscript/pict files of the figures (figures 1 and 2). If there is any thing that I can do to help please do not hesitate let me know.
Thank you for your interest in our work.
Respectfully submitted,
Dr. Genick Bar–Meir

cc: Larry Winkler
a short die casting list

encl: Documents,
Disk

He did not responded to this letter, so I sent him an additional one on December 6, 1997.

Paul M. Bralower
NADCA, Editor
9701 West Higgins Road, Suite 880
Rosemont, IL 60018-4721
Dear Mr. Bralower:

I have not received your reply to my certified mail to you dated August 20, 1997 in which I enclosed the paper "The mathematical theory of the pQ^2 diagram" authored by myself for your consideration (a cc was also sent to Larry Winkler from Hartzell). Please consider publishing my paper in the earliest possible issue. I believe that this paper is of extreme importance to the die casting field.
I understand that you have been very busy with the last exhibition and congress. However, I think that this paper deserves a prompt hearing.

I do not agree with your statement in your December 6, 1996 letter to me stating that "This paper is highly technical-too technical without a less-technical background explanation for our general readers ···. I do not believe that discounting your readers is helpful. I have met some of your readers and have found them to be very intelligent, and furthermore they really care about the die casting industry. I believe that they can judge for themselves. Nevertheless, I have yielded to your demand and have eliminated many of the mathematical derivations from this paper to satisfy your desire to have a "simple" presentation. This paper, however, still contains the essentials to be understood clearly.
Please note that I will withdraw the paper if I do not receive a reply stating your intentions by January 1, 1998, in writing. I do believe this paper will change the way pQ^2 diagram calculations are made. The pQ^2 diagram, as you know, is the central part of the calculations and design thus the paper itself is of same importance.
I hope that you really do see the importance of advancing knowledge in the die casting industry, and, hope that you will cooperate with those who have made the major progress in this area.
Thank you for your consideration.
Sincerely,
Dr. Genick Bar-Meir

cc: Boxter, McClimtic, Scott, Wilson, Holland, Behler, Dupre, and some other NADCA members

ps: You probably know by now that Garber's model is totally and absolutely wrong including all the other investigations that where based on it, even if they were sponsored by NADCA. (All the researchers agreed with me in the last congress)

Well that letter got him going and he managed to get me a letter in which he claim that he sent me his revisions. Well, read about it in my next letter dated January 7, 1998.
Paul M. Bralower,
NADCA, Editor
9701 West Higgins Road, Suite 880
Rosemont, IL 60018-4721
Dear Mr. Bralower:
Thank you for your fax dated December 29, 1997 in which you alleged that you sent me your revisions to my paper "The mathematical theory of the pQ^2 diagram." **I never receive any such thing!!** All the parties that got this information and myself find this paper to of extreme importance.
I did not revise my paper according to your comments on this paper, again, since I did not receive any. I decided to revised the paper since I did not received any reply from you for more than 4 months. I revised according to your comments on my previous paper on the critical slow plunger velocity. As I stated in my letter

dated December 6, 1997, I sent you the revised version as I send to all the cc list. I re–sent you the same version on December 29, 1997. Please note that this is the last time I will send you the same paper since I believe that you will claim again that you do not receive any of my submittal. In case that you claim again that you did not receive the paper you can get a copy from anyone who is on the cc list. Please be aware that I changed the title of the paper (December, 6, 1997 version) to be "How to calculate the pQ^2 diagram correctly".
I would appreciate if you respond to my e-Mail by January 14, 1998. Please consider this paper withdrawn if I will not hear from you by the mentioned date in writing (email is fine) whether the paper is accepted.
I hope that you really do see the importance of advancing knowledge in the die casting industry, and, hope that you will cooperate with those who have made the major progress in this area.
Sincerely,
Dr. Genick Bar-Meir

ps: You surely know by now that Garber's model is totally and absolutely wrong including all the other investigations that where based on it

He responded to this letter and changed his attitude ··· I thought.

January 9, 1998.

Dear Mr. Bar-Meir:
Thank you for your recent article submission and this follow-up e-mail. I am now in possession of your article "How to calculate the pQ2 diagram correctly." It is the version dated Jan. 2, 1998. I have read it and am prepared to recommend it for publication in Die Casting Engineer. I did not receive any earlier submissions of this article, I was confusing it with the earlier article that I returned to you. My apologies. However I am very pleased at the way you have approached this article. It appears to provide valuable information in an objective manner, which is all we have ever asked for. As is my policy for highly technical material, I am requesting technical personnel on the NADCA staff to review the paper as well. I certainly think this paper has a much better chance of approval, and as I said, I will recommend it. I will let you know of our decision in 2-3 weeks. Please do not withdraw it–give us a little more time to review it! I would like to publish it and I think technical reviewer will agree this time.
Sincerely,
Paul Bralower

Well I waited for a while and then I sent Mr. Bralower a letter dated Feb 2, 1998.

Dear Mr. Bralower,
Apparently, you do not have the time to look over my paper as you promise. Even a negative reply will demonstrate that you have some courtesy. But apparently the paper is not important as your experts told you and I am only a small bothering cockroach.
Please see this paper withdrawn!!!!

I am sorry that we do not agree that an open discussion on technical issues should be done in your magazine. You or your technical experts do not have to agree with my research. I believe that you have to let your readers to judge. I am sure that there is no other reasons to your decision. I am absolutely sure that you do not take into your consideration the fact that NADCA will have to stop teaching SEVERAL COURSES which are wrong according to this research.
Thank you for your precious time!!
Dr. Bar-Meir
Please note that this letter and the rest of the correspondence with you in this matter will be circulated in the die casting industry. I am sure that you stand by your decision and you would like other to see this correspondence even if they are NADCA members.

Here is the letter I received in return a letter from Paul Bralower Feb 5, 1998.

Dear Mr. Bar-Meir:
I'll have you know that you have inconvenienced me and others on our staff today with your untoward, unnecessary correspondence. If you had a working telephone or fax this e-mail would not be necessary. As it is I must reply to your letter and take it to someone else's office and have them e-mail it to you right away.
I tried to telephone you last week on Thurs. 1/29 with the news that we have agreed to publish your article, "How to Calculate the pQ^2 diagram correctly." I wanted to ask you to send the entire paper, with graphics and equations, on a disk. Because of the current status of our e-mail system, I would advise you not to e-mail it. Send it on any of the following: Syquest, Omega ZIP or Omega JAZ. Use Microsoft Office 97, Word 6.0 or Word Perfect 6.0.
The problem is I couldn't reach you by phone. I tried sending you a fax several times Thurs. and last Friday. There was no response. We tried a couple of different numbers that we had for you. Having no response, I took the fax and mailed it to you as a letter on Monday 2/2. I sent Priority 2-day Mail to your attention at Innovative Filters, 1107 16th Ave. S.E., Minneapolis, Minn, 55414. You should have received it today at latest if this address is correct for you, which it should be since it was on your manuscript.
Now, while I'm bending over backwards to inform you of your acceptance, you have the nerve to withdraw the paper and threaten to spread negative gossip about me in the industry! I know you couldn't have known I was trying to contact you, but I must inform you that I can't extend any further courtesies to you. As your paper has been accepted, I expect that you will cancel your withdrawl and send me the paper on disk immediately for publication. If not, please do not submit any further articles.

My response to Paul M. Bralower.

Feb 9, 1998

Dear Mr. Bralower:
Thank you for accepting the paper "How to calculate the pQ^2 diagram correctly". I strongly believe that this paper will enhance the understanding of your readers

on this central topic. Therefore, it will help them to make wiser decisions in this area, and thus increase their productivity. I would be happy to see the paper published in Die Casting Engineer.
As you know I am zealous for the die casting industry. I am doing my utmost to promote the knowledge and profitability of the die casting industry. I do not apologize for doing so. The history of our correspondence makes it look as if you refuse to publish important information about the critical slow plunger velocity. The history shows that you lost this paper when I first sent it to you in August, and also lost it when I resubmitted it in early December. This, and the fact that I had not heard from you by February 1, 1998, and other information, prompted me to send the email I sent. I am sure that if you were in my shoes you would have done the same. My purpose was not to insults anyone. My only aim is to promote the die casting industry to the best of my ability. I believe that those who do not agree with promoting knowledge in die casting should not be involved in die casting. I strongly believe that the editor of NADCA magazine (Die Casting Engineer) should be interested in articles to promote knowledge. So, if you find that my article is a contribution to this knowledge, the article should be published. I do not take personal insult and I will be glad to allow you to publish this paper in Die Casting Engineer. I believe that the magazine is an appropriate place for this article. To achieve this publication, I will help you in any way I can. The paper was written using LaTeX, and the graphics are in postscript files. Shortly, I will send you a disc containing all the files. I will also convert the file to Word 6.0. I am afraid that conversion will require retyping of all the equations. As you know, WORD produces low quality setup and requires some time. Would you prefer to have the graphic files to be in TIFF format? or another format? I have enhanced the calculations resolution and please be advised that I have changed slightly the graphics and text.
Thank you for your assistance.
Sincerely,
Dr. Genick Bar-Meir

Is the battle over?

Well, I had thought in that stage that the paper would finally be published as the editor had promised. Please continue to read to see how the saga continues.

4/24/98

Dear Paul Bralower:
To my great surprise you did not publish my article as you promised. You also did not answer my previous letter. I am sure that you have a good reasons for not doing so. I just would like to know what it is. Again, would you be publishing the article in the next issue? any other issue? published at all? In case that you intend to publish the article, can I receive a preprint so I can proof-read the article prior to the publication?

Thank you for your consideration and assistance!!
Genick

Then I got a surprise: the person dealing with me was changed. Why? (maybe you, the reader, can guess what the reason is). I cannot imagine if the letter was an offer to buy me out. I just wonder why he was concerned about me not submitting proposals (or this matter of submitting for publication). He always returned a prompt response to my proposals, yah sure. Could he possibly have suddenly found my research to be so important. Please read his letter, and you can decide for yourself.
Here is Mr. Steve Udvardy response on Fri, 24 Apr 1998

Genick,
I have left voice mail for you. I wish to speak with you about what appears to be non-submittal of your proposal I instructed you to forward to CMC for the 1999 call.
I can and should also respond to the questions you are posinjg to Paul.
I can be reached by phone at 219.288.7552.
Thank you,
Steve Udvardy

Since the deadline for that proposal had passed long before, I wondered if there was any point in submitting any proposal. Or perhaps there were exceptions to be made in my case? No, it couldn't be; I am sure that he was following the exact procedure. So, I then sent Mr. Udvardy the following letter.

April 28, 1998

Dear Mr. Udvardy:

Thank you very much for your prompt response on the behalf of Paul Bralower.
As you know, I am trying to publish the article on the pQ^2 diagram. I am sure that you are aware that this issue is central to die casting engineers. A better design and a significant reduction of cost would result from implementation of the proper pQ^2 diagram calculations.
As a person who has dedicated the last 12 years of his life to improve the die casting industry, and as one who has tied his life to the success of the die casting industry, I strongly believe that this article should be published. And what better place to publish it than "Die Casting Engineer"?
I have pleaded with everyone to help me publish this article. I hope that you will agree with me that this article should be published. If you would like, I can explain further why I think that this article is important.
I am very glad that there are companies who are adopting this technology. I just wish that the whole industry would do the same.
Again, thank you for your kind letter.
Genick

ps: I will be in my office Tuesday between 9-11 am central time (612) 378–2940

I am sure that Mr. Udvardy did not receive the comments of/from the referees (see Appendix B). And if he did, I am sure that they did not do have any effect on him whatsoever. Why should it have any effect on him? Anyhow, I just think that he was very busy with other things so he did not have enough time to respond to my letter. So I had to send him another letter.

5/15/98

Dear Mr. Udvardy:
I am astonished that you do not find time to answer my letter dated Sunday, April, 26 1998 (please see below copy of that letter). I am writing you to let you that there is a serious danger in continue to teach the commonly used pQ^2 diagram. As you probably know (if you do not know, please check out IFI's web site www.dieperfect.com), the commonly used pQ^2 diagram as it appears in NADCA's books violates the first and the second laws of thermodynamics, besides numerous other common sense things. If NADCA teaches this material, NADCA could be liable for very large sums of money to the students who have taken these courses. As a NADCA member, I strongly recommend that these classes be suspended until the instructors learn the correct procedures. I, as a NADCA member, will not like to see NADCA knowingly teaching the wrong material and moreover being sued for doing so.
I feel that it is strange that NADCA did not publish the information about the critical slow plunger velocity and the pQ^2 diagram and how to do them correctly. I am sure that NADCA members will benefit from such knowledge. I also find it beyond bizarre that NADCA does not want to cooperate with those who made the most progress in the understanding die casting process. But if NADCA teaching the wrong models might ends up being suicidal and I would like to change that if I can.
Thank you for your attention, time, and understanding!
Sincerely, Dr. Genick Bar-Meir

ps: Here is my previous letter.

Now I got a response. What a different tone. Note the formality (Dr Bar-Meir as oppose to Genick).

May 19, 1998

Dear Dr Bar-Meir,

Yes, I am here. I was on vacation and tried to contact you by phone before I left for vacation. During business travel, I was sorry to not be able to call during the time period you indicated.
As Paul may have mentioned, we have approved and will be publishing your article on calculating PQ2. The best fit for this is an upcoming issue dedicated to process control. Please rest assured that it will show up in this appropriate issue of DCE magazine.

Since there has been communications from you to Paul and myself and some of the issues are subsequently presented to our Executive Vice President, Dan Twarog, kindly direct all future communications to him. This will assist in keeping him tied in the loop and assist in getting responses back to you. His e-mail address is Twarog@diecasting.org.
Thank you,
Steve Udvardy

Why does Mr. Udvardy not want to communicate with me and want me to write to Executive Vice President? Why did they change the title of the article and omit the word "correctly". I also wonder about the location in the end of the magazine.

I have submitted other proposals to NADCA, but really never received a reply. Maybe it isn't expected to be replied to? Or perhaps it just got/was lost?

Open letter to Leo Baran

In this section an open letter to Leo Baran is presented. Mr. Baran gave a presentation in Minneapolis on May 12, 1999, on "Future Trends and Current Projects" to "sell" NADCA to its members. At the conclusion of his presentation, I asked him why if the situation is so rosy as he presented, that so many companies are going bankrupt and sold. I proceeded to ask him why NADCA is teaching so many erroneous models. He gave me Mr. Steve Udvardy's business card and told me that he has no knowledge of this and that since he cannot judge it, he cannot discuss it. Was he prepared for my questions or was this merely a spontaneous reaction?
Dear Mr. Baran,
Do you carry Steve Udvardy's business card all the time? Why? Why do you not think it important to discuss why so many die casting companies go bankrupt and are sold? Is it not important for us to discuss why there are so many financial problems in the die casting industry? Don't you want to make die casting companies more profitable? And if someone tells you that the research sponsored by NADCA is rubbish, aren't you going to check it? Discuss it with others in NADCA? Don't you care whether NADCA teaches wrong things? Or is it that you just don't give a damn?
I am sure that it is important for you. You claimed that it is important for you in the presentation. So, perhaps you care to write an explanation in the next NADCA magazine. I would love to read it.
Sincerely,

Genick Bar–Meir

Is it all coincidental?

I had convinced Larry Winkler in mid 1997 (when he was still working for Hartzell), to ask Mr. Udvardy why NADCA continued support for the wrong models (teaching the erroneous Garber's model and fueling massive grants to Ohio State University). He went to NADCA and talked to Mr. Udvardy about this. After he came back, he explained that they told him that I didn't approach NADCA in the right way. (what is that?) His enthusiasm then evaporated, and he continues to say that, because NADCA likes evolution and not revolution, they cannot support any of my revolutionary ideas. He suggested that I needed to learn to behave before NADCA would ever cooperate with me. I was surprised and shaken. "What happened, Larry?" I asked him. But I really didn't get any type of real response. Later (end of 1997) I learned he had received NADCA's design award. You, the reader, can conclude what happened; I am just supplying you with the facts.

Several manufacturers of die casting machines, Buler, HPM, Prince, and UBE presented their products in Minneapolis in April 1999. When I asked them why they do not adapt the new technologies, with the exception of the Buler, the response was complete silence. And just Buler said that they were interested; however, they never later called. Perhaps, they lost my phone number. A representative from one of the other companies even told me something on the order of "Yeah, we know that the Garber and Brevick models are totally wrong, but we do not care; just go away–you are bothering us!".

I have news for you guys: **the new knowledge is here to stay and if you want to make the die casting industry prosper, you should adopt the new technologies.** You should make the die casting industry prosper so that you will prosper as well; please do not look at the short terms as important.

The next issue of the Die Casting Engineer (May/Jun 1999 issue) was dedicated to machine products. Whether this was coincidental, you be the judge.

I submitted a proposal to NADCA (November 5, 1996) about Garber/Brevick work (to which I never received a reply). Two things have happened since: I made the proposal(in the proposal I demonstrate that Brevick's work from Ohio is wrong) 1) publishing of the article by Bill Walkington in NADCA magazine about the "wonderful research" in Ohio State University and the software to come. 2)a "scientific" article by EKK. During that time EKK also advertised how good their software was for shot sleeve calculations. Have you seen any EKK advertisements on the great success of shot sleeve calculations lately?

Here is another interesting coincidence, After 1996, I sent a proposal to NADCA, the cover page of DCE showing the beta version of software for calculating the critical slow plunger velocity. Yet, no software has ever been published. Why? Is it accidental that the author of the article in the same issue was Bill Walkington.

And after all this commotion I was surprised to learn in the (May/June 1999) issue of DCE magazine that one of the Brevick group had received a prize (see picture below if I get NADCA permission). I am sure that Brevick's group has made so much progress in the last year that this is why the award was given. I just want to learn what these accomplishments are.

put the picture of Brevick, Udvardy and price guy

For a long time NADCA described the class on the pQ^2 diagram as a "A close mathematical description." After I sent the paper and told them about how the pQ^2 diagram is erroneous, they change the description. Well it is good, yet they have to say that in the past material was wrong and now they are teaching something else. or

are they?
I have submitted five (5) papers to the conference (20^{th} in Cleveland) and four (4) have been rejected on the grounds well, you can read the letter yourself:
Here is the letter from Mr. Robb.

17 Feb 1999

The International Technical Council (ITC) met on January 20th to review all submitted abstracts. It was at that time that they downselected the abstracts to form the core of each of the 12 sessions. The Call for Papers for the 1999 Congress and Exposition produced 140 possible abstracts from which to choose from, of this number aproximately 90 abstracts were selected to be reviewed as final papers. I did recieve all 5 abstracts and distribute them to the appropriate Congress Chairmen. The one abstract listed in your acceptance letter is in fact the one for which we would like to review the final paper. The Congress Chairmen will be reviewing the final papers and we will be corresponding with all authors as to any changes revisions which are felt to be appropriate.
The Congress Chairmen are industry experts and it is there sole discretion as to which papers are solicited based on abstract topic and fit to a particular session. It is unfortunate that we cannot accept all abstracts or papers which are submitted. Entering an abstract does not constitue an automatic acceptance of the abstract/or final paper.
Thank you for your inquiry, and we look forward to reviewing your final paper.
Regards,
Dennis J. Robb
NADCA

I must have submitted the **worst kind** of papers otherwise. How can you explain that only 20% of my papers (1 out of 5) accepted. Note that the other researchers' ratio of acceptance on their papers is 65%, which means that other papers are three times better than mine. Please find here the abstracts and decide if you'd like to hear such topics or not. Guess which the topic NADCA chose, in what session and on what day (third day).

A Nobel Tangential Runner Design

The tangential gate element is commonly used in runner designs. A novel approach to this runner design has been developed to achieve better control over the needed performance. The new approach is based on scientific principles in which the interrelationship between the metal properties and the geometrical parameters is described.

Vacuum Tank Design Requirements

Gas/air porosity constitutes a large part of the total porosity. To reduce the porosity due to the gas/air entrainment, vacuum can be applied to remove the residual air in the die. In some cases the application of vacuum results in a high quality casting while in other cases the results are not satisfactory. One of the keys to the success is the design of the vacuum system, especially the vacuum tank. The present study deals with what are the design requirements on the vacuum system. Design criteria are presented to achieve an effective vacuum system.

How Cutting Edge technologies can improve your Process Design approach

A proper design of the die casting process can reduce the lead time significantly. In this paper a discussion on how to achieve a better casting and a shorter lead time utilizing these cutting edge technologies is presented. A particular emphasis is given on the use of the simplified calculations approach.

On the effect of runner design on the reduction of air entrainment: Two Chamber Analysis

Reduction of air entrapment reduces the product rejection rate and always is a major concern by die casting engineers. The effects of runner design on the air entrapment have been disregarded in the past. In present study, effects of the runner design characteristics are studied. Guidelines are presented on how to improve the runner design so that less air/gas are entrapped.

Experimental study of flow into die cavity: Geometry and Pressure effects

The flow pattern in the mold during the initial part of the injection is one of the parameters which determines the success of the casting. This issue has been studied experimentally. Several surprising conclusions can be drawn from the experiments. These results and conclusions are presented and can be used by the design engineers in their daily practice to achieve better casting.

Afterward

At the 1997 NADCA conference I had a long conversation with Mr. Warner Baxter. He told me that I had ruffled a lot of feathers in NADCA. He suggested that if I wanted to get real results, I should be politically active. He told me how bad the situation had been in the past and how much NADCA had improved. But here is something I cannot understand: isn't there anyone who cares about the die casting industry and who wants it to flourish? If you do care, please join me. I actually have found some individuals who do care and are supporting my efforts to increase scientific knowledge in die casting. Presently, however, they are a minority. I hope that as Linux is liberating the world from Microsoft, so too we can liberate and bring prosperity to the die casting industry.

After better than a year since my first (and unsent) letter to Steve Udvardy, I feel that there are things that I would like to add to the above letter. After my correspondence with Paul Bralower, I had to continue to press them to publish the article about the pQ^2 . This process is also described in the preceding section. You, the reader, must be the judge of what is really happening. Additionally, open questions/discussion topics to the whole die casting community are added.
What happened to the Brevick's research? Is there still no report? And does this type of research continue to be funded?
Can anyone explain to me how NADCA operates?
Is NADCA, the organization, more important than the die casting industry?

BIBLIOGRAPHY

[1] ASM. *Metals Handbook* , volume 13. ASM, Metals Park, Ohio, 1987.

[2] G. Backer and Frank Sant. Using Finite Element Simulation for the Development of Shot Sleeve Velocity Profiles . In *NADCA* 19^{th} *Congress and exposition*, Minneapolis, Minnesota, November 1997. paper T97-014.

[3] G. Bar-Meir, E.R.G. Eckert, and R. J. Goldstein. Pressure die casting: A model of vacuum pumping. *Journal of Engineering for Industry* , 118:001 – 007, February 1996.

[4] Genick Bar-Meir. *On gas/air porosity in pressure die casting* . PhD thesis, University of Minnesota, 1995.

[5] Genick Bar-Meir. Analysis of mass transfer processes in the pore free technique. *Journal of Engineering Materials and Technology* , 117:215 – 219, April 1995.

[6] A. Bochvar, A., M. Notkin, E., S. I. Spektorova, and N.M. Sadchikova. The Study of Casting Systems by Means of Models. *Izvest. Akad. Nauk U.S.S.R. (Bulletin of the Academy of Sciences of U.S.S.R)* , pages 875–882, 1946.

[7] Hydraulics Laboratory Branch. *Hydraulic Design of Stilling Basins and Bucket Energy Dissipatiors* . U.S. Bureau of Reclamation, Engineering Monograph 25, Denver, Colorado, 1958.

[8] Jerald R. Brevick, Dwaine J. Armentrout, and Yeou-Li Chu. Minimization of entrained gas porosity in aluminum horizontal cold chamber die castings . *Transactions of NAMRI/SME* , 12:41–46, November - December 1994.

[9] Derek L. Cocks. *DCRF Recommended Procedures: Metal Flow Predictor System*. American Die Casting Institute, Inc., Des Plaines, Illinois, 1986.

[10] Derek L. Cocks and A. J. Wall. Technology transfer in the united kingdom: Progress and prospects. In *Transactions* 12^{th} *International SDCE*, Minneapolis, Minnesota, 1983. paper G-T83-074.

[11] K Davey and S. Bounds. Modeling the pressure die casting process using boundary and finite element methods. *Journal of material Processing Technology*, 63:696–700, 1997.

[12] A. J. Davis. Effects of the relationship between molten metal flow in feed systems and hydraulic fluid flow in die casting machines. In *Transactions* 8^{th} *International SDCE*, St. Louis, 1975. paper G-T75-124.

[13] E.R.G. Eckert. Similarity analysis applied to the Die Casting Process . *Journal of Engineering Materials and Technology* , 111:393–398, 1989. No. 4 Oct.

[14] Mohamed El-Mehalawi, Jihua Liu, and R. A. Miller. A cost estimating model for die cast products. In *NADCA* 19^{th} *Congress and exposition*, Minneapolis, Minnesota, NOvember 1997. paper T97-044.

[15] Fairbanks. *Hydraulic HandBook*. Mores and Co., Kansas city, Kansas, 1959.

[16] H. Fondse, H. Jeijdens, and G Ooms. On the influence of the exit conditions on the entrainment rate in the development region of a free, round, turbulent jet. *Applied Scientific Research*, pages 355–375, 1983.

[17] L. W. Garber. Theoretical analysis and experimental observation of air entrapment during cold chamber filling. *Die Casting Engineer* , 26 No. 3:33, May - June 1982.

[18] G. Hansen, Arthur. *Fluid Mechanics*. John Wiley and Sons, Inc., New York, New York, 1967.

[19] F. M. Henderson. *Open Channel Flow*. Macmillan Publishing Co., New York, New York, 1966.

[20] A. G. Horacio and R. A. Miller. Die casting die deflections: computer simulation of causes and effects. In *NADCA* 19^{th} *Congress and exposition*, Minneapolis, Minnesota, NOvember 1997. paper T97-023.

[21] Henry Hu and S. A. Argyropoulos. Mathematical modeling of solidification and melting: a review . *Modeling Simulation Mater. Sci. Eng.* , 4:371–396, 1996.

[22] J. Hu, S. Ramalingam, G. Meyerson, E.R.G. Eckert, and R. J. Goldstein. Experiment and computer modeling of the flows in pressure die casting casings. In *ASME/CIE Design*, San Francisco, California, 1992.

[23] C. M. Kim and Frank J. Sant. An application of 3–D solidification analysis to large complex castings. In 2^{nd} *Pacific rim international conference on modeling of casting and solidification*, Singapore, January 1995.

[24] P.A. Madsen and Svendsen. Turbulent bores and hydraulic jumps. *Journal of Fluid Mechanics*, 129:1–25, 1983.

[25] Ralph David Maier. *Influence of liquid metal jet character on heat transfer during die casting*. PhD thesis, Case Western Reserve University, 1974. Engineering, metallurgy.

[26] T Nguyen and J. Carrig. Water Analogue Studies of Gravity Tilt Casting Copper Alloy components. *AFS Trans* , pages 519–528, 1986.

[27] Richard H. F. Pao. *Fluid Mechanics*. John Wiley and Sons, Inc., New York, New York, 1961.

[28] N. Rajaratnam. The hydraulic jump as a wall jet. *Journal of Hydraulic Div. ASCE*, pages 107–131, 1965. 91 (HY5).

[29] Ascher H. Shapiro. *The Dynamics and thermodynamics of Compressible Fluid Flow* , volume I. John Wiley and Sons, New York, 1953.

[30] W. F. Stuhrke and J. F. Wallace. Gating of die castings. *Transactions of American Foundrymen's Society* , 73:569–597, 1966.

[31] Marilyn Thome and Jerald R. Brevick. Optimal slow shot velocity profiles for cold chamber die casting. In *NADCA Congress and exposition*, Indianapolis, Indiana, October 1995. paper T95-024.

[32] I Wygnanski and F. H. Champan. The origin of puffs and slugs and the flow in a turbulent slug. *J. Fluid Mechanics* , pages 281–335, 1973.

Subjects index

Authors index